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The $\mu \to e\gamma$ and $\tau \to \mu\gamma$ decays with the inclusion of Lorentz and CPT violating effects

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Abstract: We study the Lorentz and CPT violating effects on the branching ratio and the CP violating asymmetry of the lepton flavor violating interactions $\mu \to e\gamma$ and $\tau \to \mu\gamma$, in the model III version of the two Higgs doublet model. Here we consider that the Lorentz and CPT violating effects exist in the QED part of the interactions and enter into expressions in the lepton propagators and in the lepton-photon vertex. We observe that there exists a non-zero CP asymmetry. However, the Lorentz and CPT violating effects on the braching ratio and the CP asymmetry are negligibly small.

Keywords: Non-Commutative Geometry, Standard Model, Rare Decays.

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Contents

1. Introduction

The Lorentz and CPT symmetries are conserved in the standard model (SM). At higher scales, like the Planck scale, there are signals that these symmetries are broken [1], for instance in the string theory [2], in the non-commutative theories [3]. In [4] it was emphasized that the spacetime-varying coupling constants can be associated with violations of local Lorentz invariance and CPT symmetry. The discussions on CPT violation on neutrino oscillations are presented in [5].

At the low energy level, the small violations of these symmetries can appear and with the inclusion of them the general Lorentz and CPT violating extension of the SM is obtained [6, 7]. In the extension of the SM the Lorentz and CPT violating effects are carried by the coefficients coming from an underlying theory at the Planck scale [2, 3]. There are various studies on the bounds of these coefficients in the literature. They have been constrained by the experiments involving hadrons [8]–[10], protons and neutrons [11], electrons [12, 13], photons [14], muons [15]. The natural suppression scale for these coefficients can be taken as the ratio of the light one m_l to the one of the order of the Planck mass . Therefore the coefficients which carry the Lorentz and CPT violating effects are in the the range of $10^{-23} - 10^{-17}$ [13]. Here the first (second) number represent the electron mass m_e ($m_{EW} \sim 250\,\mathrm{GeV}$) scale.

In the recent work [16] the amplitude for vacuum photon splitting in the framework of general Lorentz and CPT violating Quantum Electro Dynamics (QED) extensions have been analyzed and it was observed that radiative corrections arising from Lorentz violation in the fermion sector induce the vacuum photon splitting. In [17] the one loop renormalizability of the general Lorentz and CPT violating extension QED has been showed.

In the present work we study the Lorentz and CPT violating effects on the branching ratio (BR) and the CP violating asymmetry A_{CP} for the lepton flavor violating (LFV) interactions $\mu \to e\gamma$ and $\tau \to \mu\gamma$, in the model III version of the two Higgs doublet model (2HDM), since these decays do not exist in the SM. Here we consider that the Lorentz and CPT violating effects exist in the QED part of the interactions and enter into expressions in the lepton propagators and the lepton-photon vertex.

In the literature, there are several studies on LFV interactions in different models. Such interactions are analyzed in a model independent way in [18], in the framework of model III 2HDM [19]–[21], in supersymmetric models [22]–[28]. Furthermore the experimental current limits for the BR's ratios of the processes $\mu \to e\gamma$ and $\tau \to \mu\gamma$ are 1.2×10^{-11} [29] and 1.1×10^{-6} [30] respectively.

The inclusion of the Lorentz and CPT violating effects in the model III does not bring a detectable correction to the BR of the LFV processes under consideration, since the corresponding coefficients are highly suppressed at the low energy scale. However we try to examine the relative importance of the different coefficients which switch on the Lorentz and CPT violating effects in the BR of the decays $\mu \to e\gamma$ and $\tau \to \mu\gamma$. In addition to this, we analyze the possible A_{CP} in these decays and the coefficients which are sources of A_{CP}. Notice that the A_{CP} does not exist for the LFV decays $l_1 \to l_2\gamma$ in the framework of the model III and the possibility of such asymmetry has been studied in [20]. In this work it was assumed that the A_{CP} could be switched on when one considered the model beyond the model III and insert a new parameter into the interactions. The magnitude of the A_{CP} is directly proportional to this new parameter. The inclusion of the Lorentz and CPT violating effects in the model III causes a non-zero A_{CP}, however it is too small to be detected, similar to the corrections on the BRs of these processes.

The paper is organized as follows: In section 2, we present the theoretical expression for the matrix element and the A_{CP} of LFV interaction $l_1 \to l_2\gamma$ with the inclusion of the Lorentz and CPT violating effects. Section 3 is devoted to discussion and our conclusions. In the appendix, we present the explicit forms of the functions appearing in the calculation of the matrix element of the decays under consideration.

2. The LFV interactions $\mu \to e\gamma$ and $\tau \to \mu\gamma$ with the addition of Lorentz and CPT violating effects

This section is devoted to the derivation of the Lorentz and CPT violating effects on the BR and the CP asymmetry of the LFV $l_1 \to l_2\gamma$ decay. The LFV interactions in the tree level are allowed in the general 2HDM, the so-called model III and the the LFV process can be regulated by the Yukawa interaction,

$$\mathcal{L}_Y = \eta^E_{ij} \bar{l}_{iL} \phi_1 E_{jR} + \xi^E_{ij} \bar{l}_{iL} \phi_2 E_{jR} + \text{h.c.}\,, \tag{2.1}$$

where i, j are family indices of leptons, L and R denote chiral projections $L(R) = 1/2(1 \mp \gamma_5)$, ϕ_i for $i = 1, 2$, are the two scalar doublets, l_{iL} and E_{jR} are lepton doublets and singlets respectively. Here ϕ_1 and ϕ_2 are chosen as

$$\phi_1 = \frac{1}{\sqrt{2}} \left[\begin{pmatrix} 0 \\ v + H^0 \end{pmatrix} + \begin{pmatrix} \sqrt{2}\chi^+ \\ i\chi^0 \end{pmatrix} \right] ; \qquad \phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}H^+ \\ H_1 + iH_2 \end{pmatrix} , \tag{2.2}$$

where only ϕ_1 has a vacuum expectation value;

$$\langle \phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} ; \qquad \langle \phi_2 \rangle = 0\,. \tag{2.3}$$

JHEP06(2003)016

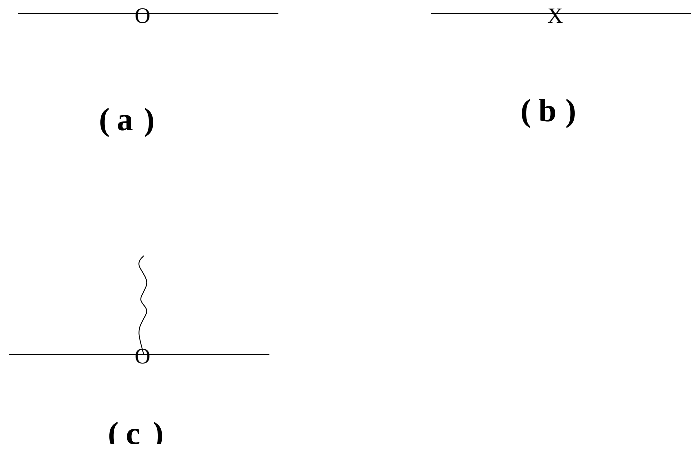

Figure 1: The CPT and Lorentz violating insertions. (a) Lepton propagator insertion ($= i\,\Gamma_1^{\mu} p_{\mu}$ where p is the 4-momentum vector of the internal lepton). (b) Lepton propagator mass insertion ($= -i\,m_1$). (c) Lepton-photon insertion ($= -iq\,\Gamma_1^{\mu}$ where q is the lepton charge).

Now we consider the gauge and CP invariant Higgs potential which spontaneously breaks $\mathrm{SU(2)} \times \mathrm{U(1)}$ down to $\mathrm{U(1)}$ as:

$$\begin{aligned} V(\phi_1, \phi_2) &= c_1 \left(\phi_1^+ \phi_1 - \frac{v^2}{2}\right)^2 + c_2 (\phi_2^+ \phi_2)^2 + \\ &+ c_3 \left[\left(\phi_1^+ \phi_1 - \frac{v^2}{2}\right) + \phi_2^+ \phi_2\right]^2 + c_4 [(\phi_1^+ \phi_1)(\phi_2^+ \phi_2) - (\phi_1^+ \phi_2)(\phi_2^+ \phi_1)] + \\ &+ c_5 [\,\mathrm{Re}(\phi_1^+ \phi_2)]^2 + c_6 [\,\mathrm{Im}(\phi_1^+ \phi_2)]^2 + c_7\,, \end{aligned} \tag{2.4}$$

with constants c_i, $i = 1, \ldots, 7$. With the choice of ϕ_1, ϕ_2 and the potential $V(\phi_1, \phi_2)$, H_1 and H_2 are obtained as the mass eigenstates h^0 and A^0 respectively, since no mixing occurs between two CP-even neutral bosons H^0 and h^0 in the tree level.

The FCNC is produced by the part of the lagrangian

$$\mathcal{L}_{Y,FC} = \xi^{E}_{ij} \bar{l}_{iL} \phi_2 E_{jR} + \mathrm{h.c.} \tag{2.5}$$

Here the Yukawa matrices ξ^E_{ij} are responsible for the LFV interactions and, in general, they have complex entries. Notice that in the following we replace ξ^E with ξ^E_N where "N" denotes the word "neutral".

At this stage we insert the Lorentz and CPT violating effects with the assumption that they exist in the QED part of the interactions. The fermionic part of the general Lorentz and CPT violating QED lagrangian in 4 space-time dimensions reads [6]

$$L = \frac{i}{2} \bar{\psi} \Gamma^{\mu} D_{\mu} \psi - \bar{\psi} M \psi\,, \tag{2.6}$$

where

$$\begin{aligned} \Gamma^{\mu} &= \gamma^{\mu} + \Gamma_1^{\mu} \\ M &= m + m_1 \end{aligned} \tag{2.7}$$

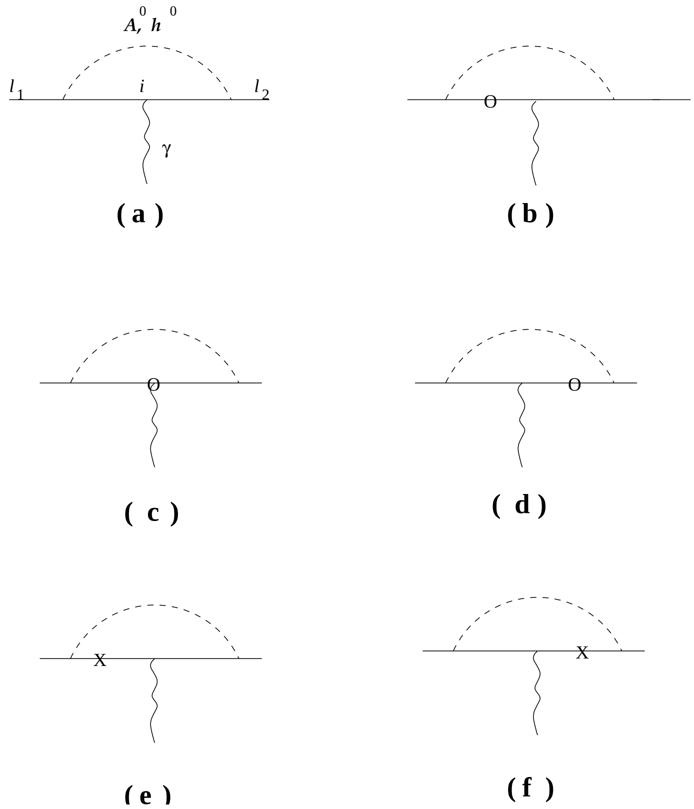

Figure 2: One loop diagrams contribute to the LFV interactions $l_1 \to l_2\gamma$ due to the neutral Higgs bosons h_0 and A_0 in the 2HDM. Dashed (curly, straight) lines represent h_0 and A_0 fields (electromagnetic field, lepton), the signs O and $\times$ represent the insertions into the propagator and the photon-fermion vertex.

and

$$
\begin{aligned}
\Gamma_1^{\mu} &= c^{\alpha\mu}\,\gamma_{\alpha} + d^{\alpha\mu}\,\gamma_5\,\gamma_{\alpha} + e^{\mu} + i f^{\mu}\,\gamma_5 + \frac{1}{2}\,g^{\lambda\nu\mu}\,\sigma_{\lambda\nu}\,, \\
m_1 &= a_{\mu}\,\gamma^{\mu} + b_{\mu}\gamma_5\,\gamma^{\mu} + \frac{1}{2}\,h_{\mu\nu}\sigma^{\mu\nu}\,. \qquad (2.8)
\end{aligned}
$$

Here the coefficients a_{μ}, b_{μ}, $c_{\alpha\mu}$, $d_{\alpha\mu}$, e_{μ}, f_{μ}, $g_{\lambda\nu\mu}$ and $h_{\mu\nu}$ cause the Lorentz violation. Among them a_{μ}, b_{μ}, e_{μ}, f_{μ} and $g_{\lambda\nu\mu}$ are responsible for the CPT violation (see [17] for details).

The LFV $l_1 \to l_2\gamma$ interaction occurs with the help of the neutral Higgs bosons, namely, Higgs bosons h_0 and A_0, in the model III. The Lorentz and CPT violating effects in Γ_1^{μ} and m_1 (see eq. 2.8) are taken into account with the insertions in the internal lepton propagator and the additional fermion-photon vertex (see figure 1). Now we use the on-shell renormalization scheme to calculate the matrix element for the LFV process under consideration. Since, in the on-shell renormalization scheme, the self energy $\sum(p)$ can be written as

$$
\sum(p) = (\hat{p} - m_{l_2})\bar{\sum}(p)(\hat{p} - m_{l_1})\,, \qquad (2.9)
$$

with $\hat{p} = \gamma_\mu p^\mu$, the corresponding diagrams vanish when $l_1(l_2)$-lepton is on-shell. However, the vertex diagrams (figure 2) give a non-zero contribution and the logarithmic divergences are eliminated by inserting an appropriate the counter term Γ^C

$$\Gamma_\mu^{\mathrm{Ren}} = \Gamma_\mu^0 + \Gamma_\mu^C \,, \tag{2.10}$$

where $\Gamma_\mu^{\mathrm{Ren}}$ and Γ_μ^0 are renormalized and bare vertex functions respectively. Here the $\Gamma_\mu^{\mathrm{Ren}}$ satisfies the equation,

$$k^\mu \, \Gamma_\mu^{\mathrm{Ren}} = 0 \,, \tag{2.11}$$

where k is the 4-momentum vector of outgoing photon. Now, the matrix element of the LFV $l_1 \to l_2 \gamma$ process with the addition of the Lorentz and CPT violating effects is obtained as,

$$M = \frac{\sqrt{4\pi\alpha_e}}{32\pi^2} \left\{ A\gamma_\mu + B_{\mu\alpha}\gamma_\alpha + iH_\alpha\sigma_{\mu\alpha} + E_\mu + A'\gamma_\mu\gamma_5 + B'_{\mu\alpha}\gamma_\alpha\gamma_5 + H'_\alpha\sigma_{\mu\alpha}\gamma_5 + E'_\mu\gamma_5 \right\} . \tag{2.12}$$

where $\alpha_e = 1/137$ and

$$\begin{aligned}
A &= \sum_{i=e,\mu,\tau} Q_i \, \xi_{N,il_1} \, \xi_{N,il_2} \int_0^1 dx \int_0^{1-x} dy \, (1-x-y) \, A_i(x,y) \,, \\
B_{\mu\alpha} &= \sum_{i=e,\mu,\tau} Q_i \, \xi_{N,il_1} \xi_{N,il_2} \int_0^1 dx \int_0^{1-x} dy \, (1-x-y) \, B_{i\,\mu\alpha}(x,y) \,, \\
H_\alpha &= \sum_{i=e,\mu,\tau} Q_i \, \xi_{N,il_1} \xi_{N,il_2} \int_0^1 dx \int_0^{1-x} dy \, (1-x-y) \, H_{i\,,\alpha}(x,y) \,, \\
E_\mu &= \sum_{i=e,\mu,\tau} Q_i \, \xi_{N,il_1} \xi_{N,il_2} \int_0^1 dx \int_0^{1-x} dy \, (1-x-y) \, E_{i\,,\mu}(x,y) \,, \\
A' &= \sum_{i=e,\mu,\tau} Q_i \, \xi_{N,il_1} \xi_{N,il_2} \int_0^1 dx \int_0^{1-x} dy \, (1-x-y) \, A'_i(x,y) \,, \\
B'_{\mu\alpha} &= \sum_{i=e,\mu,\tau} Q_i \xi_{N,il_1} \xi_{N,il_2} \int_0^1 dx \int_0^{1-x} dy \, (1-x-y) \, B'_{i\,\mu\alpha}(x,y) \,, \\
H'_\alpha &= \sum_{i=e,\mu,\tau} Q_i \, \xi_{N,il_1} \xi_{N,il_2} \int_0^1 dx \int_0^{1-x} dy \, (1-x-y) \, H'_{i\,,\alpha}(x,y) \,, \\
E'_\mu &= \sum_{i=e,\mu,\tau} Q_i \, \xi_{N,il_1} \xi_{N,il_2} \int_0^1 dx \int_0^{1-x} dy \, (1-x-y) \, E'_{i\,,\mu}(x,y) \,,
\end{aligned} \tag{2.13}$$

and

$$\begin{aligned}
H_{i\,,\alpha}(x,y) &= C_{i\,,\alpha}(x,y) + D_i(x,y) \, k_\alpha \,, \\
H'_{i\,,\alpha}(x,y) &= C'_{i\,,\alpha}(x,y) + D'_i(x,y) \, k_\alpha \,.
\end{aligned} \tag{2.14}$$

Here l_1 (l_2) is incoming (outgoing) leptons, i, Q_i denote the internal leptons ($i = e, \mu, \tau$), their charges and we choose the Yukawa coupling $\xi_{N,il_{1(2)}}$ real. In eqns. (2.13) and (2.14) the coefficients $A_i^{(\prime)}(x,y)$, $B_{i\,\mu\alpha}^{(\prime)}(x,y)$, $C_{i\,,\alpha}^{(\prime)}(x,y)$, $D_i^{(\prime)}(x,y)$, $E_{i\,,\mu}^{(\prime)}(x,y)$ are given in the appendix.

Finally, using the well known expression,

$$
d\Gamma = \frac{(2\,\pi)^4}{2\,m_{l_1}}\,|M|^2\,\delta^4\left(p-\sum_{i=1}^{2}p_i\right)\prod_{i=1}^{2}\frac{d^3p_i}{(2\pi)^3\,2E_i}\,, \tag{2.15}
$$

the decay width Γ is obtained in the l_1 lepton rest frame. Here p (p_i, i=1,2) is four momentum vector of l_1 lepton, (l_2 lepton, outgoing k photon).

At this stage, we calculate the CP asymmetry A_{CP} of the process $l_1 \to l_2\gamma$. In the model III the CP violation does not exist even with the choice of complex Yukawa couplings $\xi_{N,ij}$. However the addition of the Lorentz and CPT violating terms switch on the CP violating effects and these effects are very small since they are proportional to the the Lorentz and CPT violating coefficients. Using the definition of the CP asymmetry A_{CP}

$$
A_{CP} = \frac{\Gamma-\bar{\Gamma}}{\Gamma+\bar{\Gamma}}\,, \tag{2.16}
$$

where $\bar{\Gamma}$ denotes the decay width for the CP conjugate process, we get

$$
A_{CP} = \frac{\int_0^1 dx\int_0^{1-x}dy\,\Omega(x,y)}{\int_0^1 dx\int_0^{1-x}dy\,W(x,y)}\,, \tag{2.17}
$$

where

$$
\Omega(x,y) = \Omega_g(x,y)+\Omega_a(x,y)+\Omega_b(x,y)+\Omega_e(x,y) \tag{2.18}
$$

with

$$
\begin{aligned}
\Omega_g(x,y) &= -\frac{4\pi\alpha_e}{512\pi^4}\sum_{i=e,\mu,\tau} m_i m_{l_1}^4\,Q_i^2|\xi_{N,il_2}|^2|\xi_{N,il_1}|^2 sin2\theta_{il_1}g[p_0,k,k](F(z_A)-F(z_h))\times\\
&\quad\times(1-x-y)\,G_1(x,y)\,,\\
\Omega_a(x,y) &= -\frac{4\pi\alpha_e}{384\,\pi^4}\sum_{i=e,\mu,\tau} m_{l_1}^4\,Q_i^2\,|\xi_{N,il_2}|^2\,|\xi_{N,il_1}|^2\,cos^2\theta_{il_1}\,a[p_0]\,(H_1(z_h)-H_2(z_A))\times\\
&\quad\times(1-x-y)\,G_2(x,y)\,,\\
\Omega_b(x,y) &= -\frac{4\pi\alpha_e}{384\,\pi^4}\sum_{i=e,\mu,\tau} m_{l_1}^4\,m_i\,Q_i^2\,|\xi_{N,il_2}|^2\,|\xi_{N,il_1}|^2\,cos^2\theta_{il_1}\,b[p_0]\,(H_1(z_h)-H_2(z_A))\times\\
&\quad\times(1-x-y)\,G_3(x,y)\,,\\
\Omega_e(x,y) &= -\frac{4\pi\alpha_e}{384\,\pi^4}\sum_{i=e,\mu,\tau} m_{l_1}^4\,Q_i^2\,|\xi_{N,il_2}|^2\,|\xi_{N,il_1}|^2\,cos^2\theta_{il_1}\,e[p_0]\,(H_1(z_h)-H_2(z_A))\times\\
&\quad\times(1-x-y)\,G_4(x,y)\,,
\end{aligned} \tag{2.19}
$$

Here the function $F(z_S)$, $H_i(x,y)$ and $G_i(x,y)$ read

$$
\begin{aligned}
F(z_S) &= \frac{1}{m_S^2\,(z_S-1)^3}\,(3-4\,z_S+z_S^2+2\,\ln z_S)\,,\\
H_1(z_S) &= \frac{1}{m_S^2\,(z_S-1)^4}\,((z_S-1)\,\left(6\,m_i\,(3-4\,z_S+z_S^2)+m_{l_1}\,(-4+5\,z_S+5\,z_S^2)\right)+\\
&\quad+6\,(2\,m_i\,(z_S-1)+m_{l_1}\,(1-2\,z_S)\,z_S)\,\ln z_S)\,,
\end{aligned}
$$

JHEP06(2003)016

$$
\begin{aligned}
H_2(z_S) &= -6\, m_i\, F(z_S)\,, \\
G_1(x,y) &= \left(\frac{1}{L_{1,h^0}} - \frac{1}{L_{1,A^0}}\right) x\, y - \left(\frac{1}{L_{2,h^0}} - \frac{1}{L_{2,A^0}}\right) x\,(x+y-2) + \\
&\quad + 2\left(\frac{1}{L^2_{2,h^0}} + \frac{1}{L^2_{2,A^0}}\right) m_i\, m_{l_1}\, x^2\,(x+y)\,, \\
G_2(x,y) &= \frac{m_i + m_{l_1}\, x}{L^2_{2,h^0}} - \frac{m_i - m_{l_1}\, x}{L^2_{2,A^0}}\,, \\
G_3(x,y) &= \frac{x\,(1-4\,x-4\,y)}{4}\left(\frac{1}{L^2_{1,h^0}} - \frac{1}{L^2_{1,A^0}}\right), \\
G_4(x,y) &= \frac{x\,(3-4\,x-4\,y)}{2}\left(\frac{1}{L_{2,h^0}} - \frac{1}{L_{2,A^0}} + \frac{1}{L_{1,h^0}} - \frac{1}{L_{1,A^0}}\right) - \\
&\quad - x\left(\frac{f_1^+(x,y)}{L^2_{2,A^0}} + \frac{f_1^-(x,y)}{L^2_{2,h^0}} + \frac{f_2^+(x,y)}{L^2_{1,A^0}} + \frac{f_2^-(x,y)}{L^2_{1,h^0}}\right), \qquad (2.20)
\end{aligned}
$$

and

$$
\begin{aligned}
f_1^+(x,y) &= 2\, m_i\, m_{l_1}\, x + m_i^2\,(-1+x-y) - m_{l_1}^2\, x\, y\,(-1+x+y)\,, \\
f_1^-(x,y) &= 2\, m_i\, m_{l_1}\, x + m_i^2\,(1-x+y) + m_{l_1}^2\, x\, y\,(-1+x+y)\,, \\
f_2^+(x,y) &= 2\, m_i\, m_{l_1}\, x + m_i^2\,(-1+x-y) + m_{l_1}^2\, x\,(x+y)\,(-1+x+y)\,, \\
f_2^-(x,y) &= 2\, m_i\, m_{l_1}\, x + m_i^2\,(1-x+y) + m_{l_1}^2\, x\,(x+y)\,(1-x-y)\,. \qquad (2.21)
\end{aligned}
$$

Notice that the index $k = 1, 2, 3$; the parameters $L_{(1,2),S}$, z_S are given in the appendix and we do not give the explicit expression for the function $W(x, y)$ since it is very long. Here we take the Yukawa couplings ξ_{N,il_1} complex with the parametrization

$$
\xi^E_{N,il_1} = |\xi^E_{N,il_1}|\, e^{i\theta_{il_1}}\,, \qquad (2.22)
$$

and ξ_{N,il_2} real. In eq. (2.19), the coefficients $g[p_0, k, k] = m_{l_1}\, g[0, k, k]$, $a[p_0] = m_{l_1}\, a[0]$, $b[p_0] = m_{l_1}\, b[0]$ and $e[p_0] = m_{l_1}\, e[0]$ exist since we study in the rest frame of the incoming lepton l_1. Here $g[0, k, k]$ is CP even and the others, $a[0], b[0], e[0]$, are CP odd coefficients appearing in the lagrangian eq. (2.6) (see [1]). The A_{CP} is nonzero due to the complex nature of the couplings for the part Ω_g and CP odd nature of the coefficients for the part $\Omega_a + \Omega_b + \Omega_e$.

3. Discussion

In this section we analyze the Lorentz and CPT violating effects on the BR and the A_{CP} for the LFV $\mu \to e\gamma$ and $\tau \to \mu\gamma$ decays in the framework of the model III. The Yukawa couplings $\xi^E_{N,i\mu}$ and $\xi^E_{N,i\tau}$ ($i = e, \mu, \tau$) are responsible for the LFV decays. They are the free parameters of the theory and they should be restricted by respecting the appropriate experimental measurements. Fortunately, the strength of the Yukawa couplings $\xi^E_{N,ij}$ are considered as proportional to the masses of the leptons which are given by the indices and therefore, the contribution of the couplings related to the heavy leptons are dominant. For the decays $\mu \to e\gamma$ and $\tau \to \mu\gamma$ the main contribution comes from the internal τ lepton and the Yukawa couplings $\xi^{E}_{N,\tau\mu}$, $\xi^{E}_{N,\tau e}$ and $\xi^{E}_{N,\tau\tau}$, $\xi^{E}_{N,\tau\mu}$ play the main role respectively. There are various studies on the strength of these couplings in the literature. The upper limit of the coupling $\xi^{E}_{N,\tau\mu}$ has been predicted as ~ 0.15, by using experimental result of anomalous magnetic moment of muon in [31]. In [32] the coupling $\xi^{E}_{N,\tau\mu}$ and $\xi^{E}_{N,\tau\tau}$ has been estimated at the order of 0.03 and 0.15 respectively. For the coupling $\xi^{E}_{N,\tau e}$ the prediction has been done at the order of the magnitude of $10^{-4} - 10^{-3}$ in [19], by using the experimental result of the electric dipole moment of muon [33] and the upper limit of the BR of the process $\mu \to e\gamma$ [34]. Notice that the couplings $\xi^{E}_{N,ij}$ are complex in general and in the following, we use the parametrization

$$\xi^{E}_{N,ij} = \sqrt{\frac{4G_F}{\sqrt{2}}}\bar{\xi}^{E}_{N,ij}\,, \tag{3.1}$$

where $G_F = 1.6637 \times 10^{-5}(\,\mathrm{GeV}^{-2})$ is the fermi constant.

A possible small violations of Lorentz and CPT symmetry in the extension of the SM arise and those effects could be detected in the existing experiments [1]. In the present work we assume that these effects are only due to the QED part of the interactions and we take their effects into account. Even if their contributions are negligible in our processes, we try to understand the relative behavior of different coefficients, violating Lorentz and CPT symmetry, in the BR and the CP violating asymmetry A_{CP}. In our calculations we take the numerical values of the coefficients at the order of the magnitude of $10^{-20} - 10^{-18}$ respecting the existing results [17]

$$\begin{aligned} |a|, |b| &\sim m_\mu(m_\tau)\frac{m_{EW}}{M_P} < 10^{-18}\,(10^{-17})\,\mathrm{GeV}\,, \\ |d|, |c|, |e|, |f|, |g| &\sim \frac{m_\mu\,(m_\tau)}{M_P} < 10^{-21}\,(10^{-20})\,, \end{aligned} \tag{3.2}$$

where m_{EW} (M_P) is the electro weak (Planck mass) scale.

In figure 3, we present the magnitude of the coefficient dependence of the Lorentz violating part of the BR for the decay $\mu \to e\gamma$, for the real Yukawa couplings, $\bar{\xi}^{E}_{N,\tau\mu} = 30\,\mathrm{GeV}$, $\bar{\xi}^{E}_{N,\tau e} = 0.001\,\mathrm{GeV}$. Here solid (dashed, small dashed,dotted, dot-dashed) line represents the dependence to the coefficient $|a.p|$, $|b.p|$, c^{Sym}, (d^{Sym}, $|f.p|$, $|g[p,\beta,\beta]|$), $|e.p|$), in the case that the other coefficients have the same numerical value 10^{-20}. The BR is at the order of the magnitude of $10^{-33} - 10^{-30}$, which is a negligible quantity compared to the current experimental limits 10^{-11} [29]. It increases with the increasing values of the coefficients, especially $|f|$ and $|a|$. The BR is much more sensitive to the coefficient $|f|$ compared to the others.

Figure 4 is devoted to the magnitude of the coefficient dependence of the Lorentz violating part of the BR for the decay $\tau \to \mu\gamma$, for the real Yukawa couplings, $\bar{\xi}^{E}_{N,\tau\tau} = 100\,\mathrm{GeV}$, $\bar{\xi}^{E}_{N,\tau\mu} = 30\,\mathrm{GeV}$. Here solid (dashed, small dashed,dotted, dot-dashed) line represents the dependence to the coefficient $|a.p|$ ($|b.p|$, c^{Sym}, (d^{Sym}, $|f.p|$, $|g[p,\beta,\beta]|$), $|e.p|$), in the case that the other coefficients have the same numerical value 10^{-20}. The BR is at the order of the magnitude of $10^{-29} - 10^{-24}$, which is a negligible quantity compared to the current

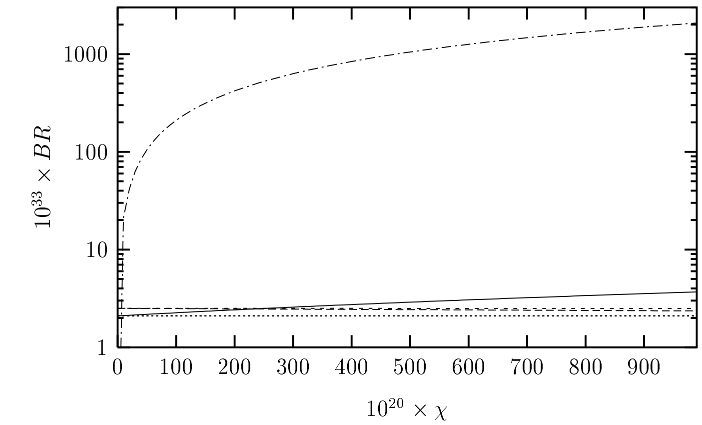

Figure 3: The magnitude χ of the coefficient dependence of the Lorentz violating part of the BR for the decay $\mu \to e\gamma$, for the real Yukawa couplings, $\bar{\xi}^{E}_{N,\tau\mu} = 30\,\mathrm{GeV}$, $\bar{\xi}^{E}_{N,\tau e} = 0.001\,\mathrm{GeV}$. Here solid (dashed, small dashed,dotted, dot-dashed) line represents the dependence to the coefficient $|a.p|$ ($|b.p|$, (c^{Sym}, d^{Sym}, $|g[p,\beta,\beta]|$), $|e.p|$, $|f.p|$), in the case that the other coefficients have the same numerical value 10^{-20}.

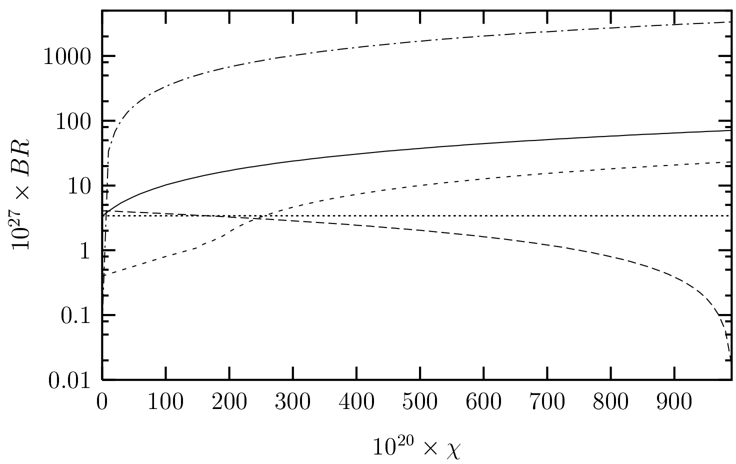

Figure 4: The same as figure 3 but for the decay $\tau \to \mu\gamma$.

limits 10^{-6} [30], similar to the previous process. Here the increasing values of the coefficients, $|f|$, $|a|$ and c^{Sym} increases the BR. However the BR decreases with the increasing values the coefficient $|b|$. The BR is much more sensitive to the coefficients $|e|$, $|a|$, $|e|$ and $|b|$ compared to the others.

Now, in figure 5 (6), we present the possible CP violating asymmetry A_{CP} for the decay $\mu \to e\gamma$ ($\tau \to \mu\gamma$), for the Yukawa couplings,$|\bar{\xi}^{E}_{N,\tau\mu}| = 30\,\mathrm{GeV}$, $|\bar{\xi}^{E}_{N,\tau e}| = 0.001\,\mathrm{GeV}$ ($|\bar{\xi}^{E}_{N,\tau\tau}| = 100\,\mathrm{GeV}$, $|\bar{\xi}^{E}_{N,\tau\mu}| = 30\,\mathrm{GeV}$), $sin\theta_{\tau\mu} = 0.5$ ($sin\theta_{\tau\tau} = 0.5$). Notice that we take $sin\theta_{\tau e} = 0$ ($sin\theta_{\tau\mu} = 0$) for the decays $\mu \to e\gamma$ ($\tau \to \mu\gamma$). Here solid (dashed, small dashed,dotted) line represents the dependence to the coefficient $|g[p_0,k,k]|$, $|a[p_0]|$, $|b[p_0]|$ and $|e[p_0]|$, in the case that the other coefficients have the same numerical value 10^{-20}. The coefficient $g[p_0,k,k]|$ ($|a[p_0]|$, $|b[p_0]|$ and $|e[p_0]|$) is CP even (odd) and the source of the A_{CP}

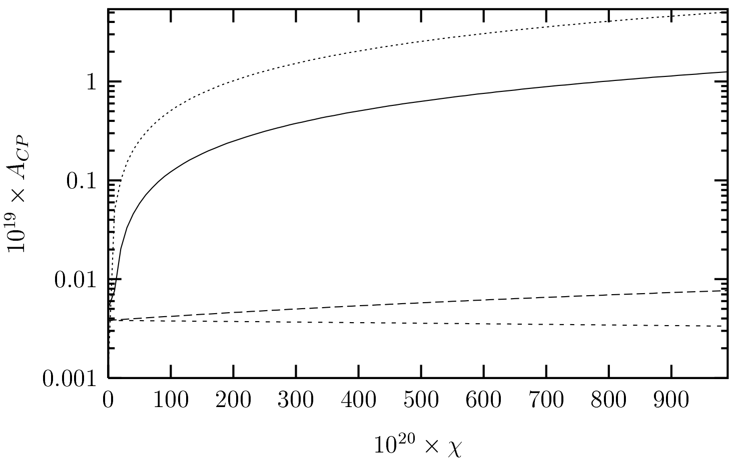

Figure 5: The magnitude χ of the coefficient dependence of the A_{CP} for the decay $\mu \to e\gamma$, for the Yukawa couplings,$|\bar{\xi}^{E}_{N,\tau\mu}| = 30\,\mathrm{GeV}$, $|\bar{\xi}^{E}_{N,\tau e}| = 0.001\,\mathrm{GeV}$, $sin\theta_{\tau\mu} = 0.5$, $sin\theta_{\tau e} = 0$. Here solid (dashed, small dashed,dotted) line represents the dependence to the coefficient $|g[p_0,k,k]|$, $|a[p_0]|$, $|b[p_0]|$ and $|e[p_0]|$, in the case that the other coefficients have the same numerical value 10^{-20}.

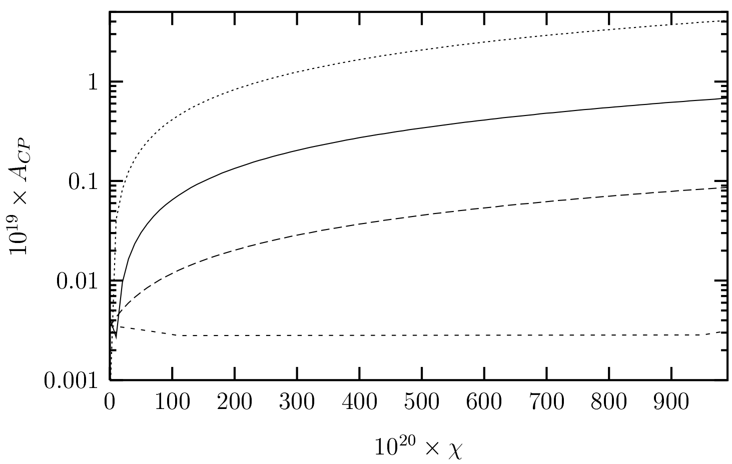

Figure 6: The magnitude χ of the coefficient dependence of the A_{CP} for the decay $\tau \to \mu\gamma$, for the Yukawa couplings,$|\bar{\xi}^{E}_{N,\tau\tau}| = 100\,\mathrm{GeV}$, $|\bar{\xi}^{E}_{N,\tau\mu}| = 30\,\mathrm{GeV}$, $sin\theta_{\tau\tau} = 0.5$, $sin\theta_{\tau\mu} = 0$. Here solid (dashed, small dashed,dotted) line represents the dependence to the coefficient $|g[p_0,k,k]|$, $|a[p_0]|$, $|b[p_0]|$ and $|e[p_0]|$, in the case that the other coefficients have the same numerical value 10^{-20}.

is the complex nature of the couplings for the part proportional to the coefficient $g[p_0,k,k]|$ and the CP odd nature of the coefficients for the part proportional to coefficients $|a[p_0]|$, $|b[p_0]|$ and $|e[p_0]|$ (see eq. (2.19)). For the $\mu \to e\gamma$ ($\tau \to \mu\gamma$) decay A_{CP} is much more sensitive to the coefficients $g[p_0,k,k]|$ and $|e[p_0]|$ ($g[p_0,k,k]|$, $|e[p_0]|$ and $|a[p_0]|$) compared to the ones $|a[p_0]|$ and $|b[p_0]|$ ($|b[p_0]|$). It increases with the increasing values of $g[p_0,k,k]|$ and $|e[p_0]|$ ($g[p_0,k,k]|$, $|e[p_0]|$ and $|a[p_0]|$). Notice that in the case of real couplings the coefficient $g[p_0,k,k]|$ does not give contribution to the A_{CP} However the numerical value of A_{CP} is very small, at the order of the magnitude of 10^{-19} for both decays and it seems that it is not possible to detect even in the future experiments.

At this stage we would like to summarize our results:

We analyse the Lorentz and CPT violating effects on the BR and A_{CP} for the LFV decays $\mu \to e\gamma$ and $\tau \to \mu\gamma$ in the framework of the model III. Here we assume that these effects are only due to the QED part of the interactions. By taking the numerical values of the coefficients at the order of the magnitude of $10^{-20} - 10^{-18}$ we study the relative behaviors of different coefficients

- The contribution of the Lorentz and CPT violating part to the BR of the decays $\mu \to e\gamma$ ($\tau \to \mu\gamma$) is at the order of the magnitude of 10^{-32} (10^{-26}), which is too small to be detected. For the decay $\mu \to e\gamma$ the BR is more sensitive to the coefficient $|e|$ compared to others and for its large values the BR reaches to the order of 10^{-30}. For the decay $\tau \to \mu\gamma$ the BR is sensitive to the coefficients $|e|$, $|a|$, $|e|$ and $|b|$ and it can take the values at the order of the magnitude of 10^{-24}

- We predict the numerical value of A_{CP} at the order of the magnitude of 10^{-19} for both decays. The source of the A_{CP} is the coefficients $g[p_0,k,k] = m_{l_1}\, g[0,k,k]$, $a[p_0] = m_{l_1}\, a[0]$, $b[p_0] = m_{l_1}\, b[0]$ and $e[p_0] = m_{l_1}\, e[0]$. Here $g[0,k,k]$ is CP even and the others, $a[0], b[0], e[0]$, are CP odd appearing in the lagrangian eq. (2.6) (see [17] for details). The A_{CP} is nonzero due to the complex nature of the couplings for the part Ω_g and CP odd nature of the coefficients for the part $\Omega_a + \Omega_b + \Omega_e$. We observe that the A_{CP} is too small to be measured even in future experiments.

A. The explicit forms of the functions appearing in the matrix element

The coefficients $A_i^{(\prime)}(x,y)$, $B_{i\,\mu\alpha}^{(\prime)}(x,y)$, $C_{i\,,\alpha}^{(\prime)}(x,y)$, $D_i^{(\prime)}(x,y)$, $E_{i\,,\mu}^{(\prime)}(x,y)$ in eqns. (2.13) and (2.14) reads

$$
\begin{aligned}
A_i(x,y) = & \left\{ \left(\frac{1}{L_{2,h^0}} + \frac{1}{L_{2,A^0}}\right)(1+x+y) + \left(\frac{1}{L^2_{2,h^0}} + \frac{1}{L^2_{2,A^0}}\right) f_1 + \right. \\
& \left. + \left(\frac{1}{L^2_{2,h^0}} - \frac{1}{L^2_{2,A^0}}\right) m_{l_1} m_i\, x(x+2y) \right\} \mathrm{a.k} + \\
& + \left\{ m_i\, y \left(\left(\frac{1}{L^2_{1,h^0}} + \frac{1}{L^2_{1,A^0}}\right) f_2 + 2\left(\frac{1}{L^2_{1,h^0}} - \frac{1}{L^2_{1,A^0}}\right) m_{l_1} m_i\, x \right) + \right. \\
& + \left(\frac{1}{L_{1,h^0}} + \frac{1}{L_{1,A^0}}\right) m_i\, y - \left((m_i + x m_{l_1}) \frac{1}{L^2_{2,h^0}} + (m_i - x m_{l_1}) \frac{1}{L^2_{2,A^0}} \right) \times \\
& \times (x+y)(m_i^2 - xy m_{l_1}^2) - \left(\left(m_i(-2+x+y) - 4\,x\,(x+y)\, m_{l_1}\right) \frac{1}{L_{2,h^0}} + \right. \\
& \left. \left. + \left(m_i(-2+x+y) + 4x(x+y) m_{l_1}\right) \frac{1}{L_{2,A^0}} \right) \right\} \mathrm{e.k} + \\
& + \left(\frac{1}{L^2_{1,h^0}} - \frac{1}{L^2_{1,A^0}}\right) m_{l_1} m_i y \left(f_3 - m_i^2 + m_{l_1}^2 x(x+y)\right) c^{\mathrm{Sym}} +
\end{aligned}
$$

JHEP06(2003)016

$$
\begin{aligned}
&+\left(\frac{1}{L_{2,h^0}^2}-\frac{1}{L_{2,A^0}^2}\right) m_{l_1}\, m_i\, x^2\,(2+x)\, c^{\rm Asym}[p,p'] + \\
&+2ix\left(\frac{1}{L_{2,h^0}^2}(m_i(x-1)-m_{l_1}xy)+\frac{1}{L_{2,A^0}^2}(m_i(x-1)+m_{l_1}xy)\right) h[p,p'] + \\
&+2\,i\left(\frac{1}{L_{1,h^0}^2}-\frac{1}{L_{1,A^0}^2}\right) m_{l_1}\, x^2\,(x+y)\,(y\, g[p,p',k]+x\, g[p,p',p]) - \\
&-i\left(x^2\left(\frac{1}{L_{1,h^0}}-\frac{1}{L_{1,A^0}}\right)+x\,(x+1)\left(\frac{1}{L_{2,h^0}}-\frac{1}{L_{2,A^0}}\right)\right) m_{l_1}\, g[k,\beta,\beta] - \\
&-i\frac{1}{L_{2,h^0}^2}\Bigg((x+y)\,x\,\left(m_i\,(1-x-y)+2\,m_{l_1}\,x\,(1-y)\right) g[p,p',k] + \\
&\qquad\qquad +\frac{1}{2}\,x^2\,\left(m_i\,(2\,x+y-2)+4\,m_{l_1}\,x\,(-1+y)\,\right)\, g[p,p',p]\Bigg) - \\
&-i\frac{1}{L_{2,A^0}^2}\Bigg((x+y)\,x\,\left(-m_i(x+y-1)+2\,m_{l_1}\,x(-1+y)\right)\, g[p,p',k] + \\
&\qquad\qquad +\frac{1}{2}\,x^2\,\left(m_i\,(2\,x+y-2)-4\,m_{l_1}\,x\,(-1+y)\,\right)\, g[p,p',p]\Bigg)\,,
\end{aligned}
$$

$$
\begin{aligned}
A'_i(x,y) \;=\; &\Bigg\{\left(\frac{1}{L_{1,h^0}^2}+\frac{1}{L_{1,A^0}^2}\right) f_4(1-x-y)-2\left(\frac{1}{L_{1,h^0}^2}-\frac{1}{L_{1,A^0}^2}\right) x(x-1)m_i m_{l_1} + \\
&+\left(\frac{1}{L_{1,h^0}}+\frac{1}{L_{1,A^0}}\right)(-1+3x-y)-\left(\frac{1}{L_{2,h^0}^2}-\frac{1}{L_{2,A^0}^2}\right) x^2\, m_i\, m_{l_1} - \\
&-\left(\frac{1}{L_{2,h^0}^2}+\frac{1}{L_{2,A^0}^2}\right)\, f_4+\left(\frac{1}{L_{2,h^0}}+\frac{1}{L_{2,A^0}}\right)\,(1+x+y)\Bigg\}\text{b.k} - \\
&-i\Bigg\{\left(\frac{1}{L_{1,h^0}^2}+\frac{1}{L_{1,A^0}^2}\right) m_i y\left(m_{l_1}^2 x(x+y)-m_i^2\right)-3m_i\left(\frac{1}{L_{1,h^0}}+\frac{1}{L_{1,A^0}}\right) - \\
&\quad-\left(\frac{m_i+m_{l_1}\,x}{L_{2,h^0}^2}+\frac{m_i-m_{l_1}\,x}{L_{2,A^0}^2}\right)(x+y)(m_i^2+x\,y\,m_{l_1}^2) - \\
&\quad-\left(\frac{3\,m_i+4\,x\,m_{l_1}}{L_{2,h^0}}+\frac{3\,m_i-4\,x\,m_{l_1}}{L_{2,A^0}}\right)\,(x+y)\,\Bigg\}\text{f.k} + \\
&+\left(\frac{1}{L_{2,h^0}^2}-\frac{1}{L_{2,A^0}^2}\right) m_{l_1}\, m_i\, x^2\,(1-3x-3y)\, d^{\rm Asym}[p,p']\frac{1}{2}\,\epsilon_{\rho\theta\alpha\beta}\,\times \\
&\times\left(\frac{1}{L_{1,h^0}^2}+\frac{1}{L_{1,A^0}^2}\right) m_i x h[\alpha,\beta] k_\rho\, p_\theta+\frac{1}{2}\,\epsilon_{\rho\theta\alpha\beta}\left(\frac{1}{L_{2,h^0}^2}+\frac{1}{L_{2,A^0}^2}\right)\times \\
&\times m_i x(1-x-y)(x+y)g[\beta,\alpha,k]k_\rho p_\theta\,,
\end{aligned}
$$

$$
\begin{aligned}
B_{i\,\mu\alpha}(x,y) \;=\; -i\Bigg\{ & \frac{1}{4}\epsilon_{\rho\alpha\beta\mu}\left(\frac{1}{L^2_{1,h^0}}-\frac{1}{L^2_{2,A^0}}\right) m_i\, m_{l_1}\, x\,(1-4\,x-4\,y)+\frac{1}{L^2_{2,h^0}}\times \\
& \times (f_4+2m_i\,(m_{l_1}x+m_i y))+\frac{1}{L^2_{2,A^0}}\,(f_4+2\,m_i\,\,(-m_{l_1}\,x+m_i\,y))+ \\
& +\left(\frac{1}{L_{2,h^0}}+\frac{1}{L_{2,A^0}}\right)\,(-1+3\,x+3\,y)\Bigg\}\,k_\rho\,b_\beta- \\
-\Bigg\{ & m_i\,m_{l_1}\,y\,\left(\left(\left(\frac{1}{L^2_{1,h^0}}-\frac{1}{L^2_{1,A^0}}\right)f_3+\left(\frac{1}{L^2_{1,h^0}}+\frac{1}{L^2_{1,A^0}}\right)m_i\,m_{l_1}\,x\right)+\right. \\
& +3\,(\frac{1}{L_{1,h^0}}-\frac{1}{L_{2,A^0}})\,m_i\,m_{l_1}\,y+m_{l_1}\,(x+y)\,(m_i^2+m_{l_1}^2\,x\,y)\times \\
& \times\left(\frac{1}{L^2_{2,h^0}}(m_i+m_{l_1}\,x)+\frac{1}{L^2_{2,A^0}}(-m_i+m_{l_1}\,x)\right)+m_{l_1}\times \\
& \times\left(\frac{1}{L_{2,h^0}}\,(4\,m_{l_1}\,x\,(x+y)+m_i\,\,(1+3\,x+3\,y))+\right. \\
& \left.\left.+\frac{1}{L_{2,A^0}}\,(4m_{l_1}x(x+y)-m_i(1+3x+3y))\right)\right\}c^{\mathrm{Asym}}[\mu,\alpha]-i\epsilon_{\rho\alpha\beta\mu}\times \\
\times\Bigg\{ & 2\left(\frac{1}{L^2_{1,h^0}}-\frac{1}{L^2_{1,A^0}}\right)m_{l_1}\,m_i\,x\,y^2+\left(\frac{1}{L^2_{2,h^0}}-\frac{1}{L^2_{2,A^0}}\right)\,m_{l_1}\,m_i\,x\,\times \\
& \times(x+y)\,(y-1)\Bigg\}\,k_\rho\,d^{\mathrm{Asym}}[\beta,k]-i\times \\
\times\Bigg\{ & \left(\frac{1}{L_{2,h^0}}-\frac{1}{L_{2,A^0}}\right)\,m_{l_1}\,(x+y)-\left(\frac{1}{L^2_{2,h^0}}-\frac{1}{L^2_{2,A^0}}\right)m_{l_1}\,f_4- \\
& -\left(\frac{1}{L^2_{2,h^0}}+\frac{1}{L^2_{2,A^0}}\right)\,m_{l_1}\,m_i\,x^2\Bigg\}\,h[\mu,\alpha]+\frac{1}{2}\epsilon_{\rho\alpha\beta\mu}\,\left(\frac{1}{L^2_{1,h^0}}-\frac{1}{L^2_{1,A^0}}\right)\times \\
\times & m_{l_1}xy^3k_\rho g[\beta,k,k]+i\Bigg\{\frac{1}{L_{1,h^0}}\,(m_i(x-1)-m_{l_1}(x+2y+2xy))+\frac{1}{L_{1,A^0}}\times \\
& \times\,(m_i\,(x-1)+m_{l_1}\,(x+2\,y+2\,x\,y))\Bigg\}\times \\
\times\, & g[k,\alpha,\mu]-2i\left(\frac{1}{L_{1,h^0}}-\frac{1}{L_{1,A^0}}\right)\,m_{l_1}\,x\,y g[p,\alpha,\mu]-\frac{i}{2}\times \\
\times\Bigg(& \frac{1}{L_{1,h^0}}\,(2\,m_i\,(x-1)+m_{l_1}\,\,(x\,(y-2)-4\,y\,\,(1-3\,y)))+\frac{1}{L_{1,A^0}}\times \\
& \times\Big(2\,m_i\,(x-1)-m_{l_1}\,(x\,(y-2)-4\,y\,(1-3\,y))\Big)\Bigg)\,g[\mu,\alpha,k]+
\end{aligned}
$$

$$
\begin{aligned}
&+i\frac{1}{2}\left(\frac{1}{L_{1,h^0}}-\frac{1}{L_{1,A^0}}\right) m_{l_1}\,x\,y\,\left(g[\mu,k,\alpha]-2\,g[\mu,p,\alpha]+10\,g[\mu,\alpha,p]\,\right)-\\
&-\frac{i}{4}\left(\frac{1}{L^2_{2,h^0}}+\frac{1}{L^2_{2,A^0}}\right) m^2_{l_1}\,m_i\,x\,\times\\
&\times\Bigg(-4\,x\,(x+y)\,g[k,\alpha,\mu]+8\,x\,(x+y)\,g[p,\alpha,\mu]+2\,\times\\
&\quad\times\left(x^2+y(y-1)+x(2y-1)\,\right)g[\mu,\alpha,k]-x(-2+2x+y)g[\mu,\alpha,p]\Bigg)+\\
&+i\left(\frac{1}{L_{2,h^0}}-\frac{1}{L_{2,A^0}}\right) m_{l_1}\,x\,(x+y)\,g[\mu,p,\alpha]+\frac{i}{2}\,\times\\
&\times\Bigg\{\frac{1}{L_{2,h^0}}\left(m_i\,(-1+x+y)+4m_{l_1}\,(x+x^2+y-y^2)\,\right)+\\
&\quad+\frac{1}{L_{2,A^0}}\left(m_i\,(-1+x+y)-4\,m_{l_1}\,(x+x^2+y-y^2)\,\right)\Bigg\}\,g[k,\alpha,\mu]-\\
&-\frac{i}{2}\,\frac{1}{L_{2,h^0}}\,\left(m_i\,(-1+x+y)+m_{l_1}\,(11\,x^2+8\,y\,(y-1)+(19\,y-8)\,x)\right)-\\
&-\frac{i}{2}\frac{1}{L_{2,A^0}}\left(m_i\,(-1+x+y)+m_{l_1}\,(-11\,x^2-8\,y\,(y-1)-(19\,y-8)\,x)\,\right)\times\\
&\times g[\mu,\alpha,k]+\frac{i}{2}\left(\frac{1}{L_{2,h^0}}-\frac{1}{L_{2,A^0}}\right)\times\\
&\times\Bigg(m_{l_1}\,\Bigg(4\,(x+y-1)\,g[p,\alpha,\mu]-(x+y)\,g[\mu,k,\alpha]+2\,(5\,x+5\,y-4)\,\times\\
&\qquad\times g[\mu,\alpha,p]\Bigg)\Bigg)\,,
\end{aligned}
$$

$$
\begin{aligned}
B'_{i\,\mu\alpha}(x,y)\;=\;&-i\epsilon_{\rho\alpha\beta\mu}\Bigg\{-\left(\frac{1}{L^2_{1,h^0}}-\frac{1}{L^2_{1,A^0}}\right) m_{l_1}m_i x+\left(\frac{1}{L^2_{1,h^0}}+\frac{1}{L^2_{1,A^0}}\right)\times\\
&\quad\times f_3(-1+x+y)+\left(\frac{1}{L_{1,h^0}}+\frac{1}{L_{1,A^0}}\right)(-1+3\,x+3\,y)+\\
&\quad+\left(\frac{1}{L^2_{2,h^0}}+\frac{1}{L^2_{2,A^0}}\right)(\,m^2_i\,(y-1)+m^2_{l_1}\,x\,y\,(-1+x+y))+\\
&\quad+\left(\frac{1}{L_{2,h^0}}+\frac{1}{L_{2,A^0}}\right)(-1+3\,x+3\,y)\Bigg\}\,k_\rho\,a_\beta-m^3_{l_1}\,m_i\,x\,\times\\
&\times\left\{2\,(\frac{1}{L^2_{1,h^0}}-\frac{1}{L^2_{1,A^0}})y^2+\left(\frac{1}{L^2_{2,h^0}}-\frac{1}{L^2_{2,A^0}}\right)(y-1)\,(x+y)\,\right\}\times\\
&\times d^{\mathrm{ASym}}[\mu,\alpha]-\epsilon_{\rho\alpha\beta\mu}\,\left(\frac{1}{L^2_{1,h^0}}+\frac{1}{L^2_{1,A^0}}\right)\,m_i\,\times
\end{aligned}
$$

$$
\times \left\{ (2\,(x-1)\,y\,h[k,\beta] + x\,(1+2\,x)\,h[p,\beta])\;k_\rho - \right.
$$

$$
\left. - \,x\;((1+2\,y)\,h[k,\beta]\,p_\rho - h[\beta,\alpha]\,k_\rho\,p_\theta + (2\,y+1)\,h[\mu,\beta]\,k_\rho\,p_\theta) \right\} -
$$

$$
-\epsilon_{\rho\theta\alpha\beta} \left\{ \frac{1}{L^2_{2,h^0}} (2\,x\,(m_i + m_{l_1}\,x)\,h[p,\beta] - (m_i\,x^2 + 2\,m_i\,y + 2m_{l_1} x^2 \times \right.
$$

$$
\times\, y + 2m_{l_1} x y^2)\,h[k,\beta]) +
$$

$$
+ \frac{1}{L^2_{2,A^0}} (2\,x\,(m_i - m_{l_1}\,x)\,h[p,\beta] - (m_i\,x^2 + 2\,m_i\,y - 2\,m_{l_1}\,x^2 \times
$$

$$
\left. \times\, y - 2\,m_{l_1}\,x\,y^2)\,h[k,\beta]) \right\} k_\rho -
$$

$$
-\,\epsilon_{\rho\alpha\beta\mu} \left\{ \left(\frac{1}{L^2_{1,h^0}} - \frac{1}{L^2_{1,A^0}} \right) (x+y)(x\,g[p,\beta,k] - 2\,y\,g[\beta,k,k] - \right.
$$

$$
-\,2\,x\,g[\beta,k,p] - x\,g[\beta,p,k]) +
$$

$$
\left. + \frac{1}{2}\,(x+y)\,m^2_{l_1}\,y^2\;\left(\frac{1}{L^2_{1,h^0}} + \frac{1}{L^2_{1,A^0}} \right)\;g[\mu,\alpha,k] \right\} k_\rho +
$$

$$
+ \frac{1}{2} \left(\frac{1}{L_{1,h^0}} - \frac{1}{L_{1,A^0}} \right) m_{l_1}\,x\,y\,\times
$$

$$
\times\,(\,\epsilon_{\alpha\beta\lambda\mu}\;(g[k,\beta,\lambda] + 2\,g[\lambda,\beta,k]\,) + \epsilon_{\rho\alpha\beta\lambda}\;(2\,g[\lambda,\beta,\mu] + g[\mu,\beta,\lambda]\,)\,k_\rho) +
$$

$$
+ \frac{1}{2}\epsilon_{\rho\alpha\beta\mu}\,(x+y) \left(\frac{1}{L^2_{2,h^0}} (m_i\,(-1+x+y) - 4m_{l_1}(y-1)(x+y)) + \right.
$$

$$
\left. + \frac{1}{L^2_{2,A^0}} \Big(m_i\,(-1+x+y) + 4\,m_{l_1}\,(y-1)\,(x+y) \Big) \right) \times
$$

$$
\times\,k_\rho\,g[\beta,k,k] - \frac{1}{4}\,\epsilon_{\rho\alpha\beta\mu}\,\times
$$

$$
\times \left\{ \frac{1}{L^2_{2,h^0}} (8\,m_{l_1}\,x\,(y-1)\,(x+y)\,(g[p,\beta,k] - g[\beta,k,p]) + m_i\,x\,\times \right.
$$

$$
\times\,(-2+2x+y)\,g[\beta,k,p]) - \frac{1}{L^2_{2,A^0}} \times
$$

$$
\times\,(8m_{l_1}\,x\,(y-1)\,(x+y)\,(g[p,\beta,k] - g[\beta,k,p]) -
$$

$$
\left. -\,m_i\,x\,(-2+2x+y)\,g[\beta,k,p]) \right\} k_\rho -
$$

$$
-\frac{1}{4} \left(\frac{1}{L^2_{2,h^0}} + \frac{1}{L^2_{2,A^0}} \right) m^2_{l_1}\,m_i\,x\,(x+y)\,\times
$$

$$
\times\,(2x\epsilon_{\alpha\beta\lambda\mu} g[\lambda,\beta,k] + \epsilon_{\rho\alpha\beta\lambda}(1+x-y)\,g[\lambda,\beta,\mu]k_\rho) - \frac{1}{2} \left(\frac{1}{L_{2,h^0}} - \frac{1}{L_{2,A^0}} \right) \times
$$

$$
\begin{aligned}
& \times x(x+y)m_{l_1}((g[k,\beta,\lambda]+2g[\lambda,\beta,k])\epsilon_{\alpha\beta\lambda\mu}+(g[\mu,\beta,\lambda]+2g[\lambda,\beta,\mu])\times \\
& \qquad\qquad \times\epsilon_{\rho\alpha\beta\lambda}k_\rho)\,, \\
C_{i\,,\alpha}(x,y) \;=\; & \left\{ m_i y\left(\left(\frac{1}{L^2_{1,h^0}}+\frac{1}{L^2_{2,A^0}}\right)m_i\,m_{l_1}x+\left(\frac{1}{L^2_{1,h^0}}-\frac{1}{L^2_{2,A^0}}\right)f_3\right)+\right. \\
& +3m_i\,y\,\left(\frac{1}{L_{1,h^0}}-\frac{1}{L_{2,A^0}}\right)+(x+y)(m_i^2+m_{l_1}^2\,x\,y\,)\times \\
& \times\left(\frac{1}{L^2_{1,h^0}}(m_i+m_{l_1}\,x)+\frac{1}{L^2_{2,A^0}}\,(-m_i+m_{l_1}\,x)\right)+ \\
& +\frac{1}{L_{2,h^0}}\,(4\,m_{l_1}\,x\,(x+y)+m_i\,(1+3\,x+3\,y))+ \\
& \left.+\frac{1}{L_{2,A^0}}\,(4\,m_{l_1}\,x\,(x+y)-m_i\,(1+3\,x+3\,y))\right\}\times \\
& \times c^{\mathrm{ASym}}[k,\alpha]-i\times \\
& \times\left\{\left(\frac{1}{L^2_{2,h^0}}+\frac{1}{L^2_{2,A^0}}\right)m_i\,m_{l_1}\,x^2+\left(\frac{1}{L^2_{2,h^0}}-\frac{1}{L^2_{2,A^0}}\right)f_4-\right. \\
& \qquad\left.-\left(\frac{1}{L_{2,h^0}}-\frac{1}{L_{2,A^0}}\right)(x+y)\right\}h[k,\alpha]+ \\
& +\frac{i}{2}\left(\frac{1}{L_{1,h^0}}-\frac{1}{L_{1,A^0}}\right)y\times \\
& \times(2\,x\,g[p,p',\alpha]+(5\,x+12\,y)\,g[k,\alpha,k]+10\,x\,g[k,\alpha,p]+4\,x\,g[\alpha,p,k]\,)- \\
& -\frac{i}{4}\,m_i\,m_{l_1}\left(\frac{1}{L^2_{1,h^0}}+\frac{1}{L^2_{1,A^0}}\right)x(x\,(-2+2\,x+y)\,g[k,\alpha,p]-2\,(x+y)\times \\
& \times((1+x-y)\,g[\alpha,k,k]-4x\,g[\alpha,p,k]))-i\left(\frac{1}{L_{1,h^0}}-\frac{1}{L_{1,A^0}}\right)x\times \\
& \times((x+y)\,g[p,p'\alpha]+(5\,x+5\,y-4)\,g[k,\alpha,p]- \\
& \left.-\left(7x^2+12y(y-1)+x(19y-12)\right)g[k,\alpha,k]-4x(-1+x+y)g[\alpha,p,k])\right\}, \\
C'_{i\,,\alpha}(x,y) \;=\; & m_i\,m^2_{l_1}\,x\left\{2\left(\frac{1}{L^2_{1,h^0}}-\frac{1}{L^2_{1,A^0}}\right)y^2+\left(\frac{1}{L^2_{2,h^0}}-\frac{1}{L^2_{2,A^0}}\right)(x+y)(y-1)\right\}\times \\
& \times d^{ASym}[k,\alpha]+3m_iy\,\left(\frac{1}{L_{1,h^0}}-\frac{1}{L_{1,A^0}}\right)c^{\mathrm{ASym}}[k,\alpha]+ \\
& +\frac{i}{2}\left(\frac{1}{L^2_{1,h^0}}-\frac{1}{L^2_{1,A^0}}\right)m^2_{l_1}\,x\,y^3\,g[\alpha,k,k]+\frac{i}{2}\left(\frac{1}{L_{1,h^0}}-\frac{1}{L_{1,A^0}}\right)\times \\
& \times(\,2\,x\,g[p,p'\alpha]-(5\,x+12\,y)\,g[\alpha,k,k]-10\,x\,g[\alpha,k,p]-4\,x\,g[\alpha,p,k])+ \\
& +\frac{1}{4}\,\epsilon_{\rho\alpha\theta\beta}\,m_i\,m_{l_1}\left(\frac{1}{L^2_{2,h^0}}+\frac{1}{L^2_{2,A^0}}\right)(x^2+y\,(y-1)+x\,(2y-1))g[\beta,\theta,k]\,k_\rho\,,
\end{aligned}
$$

$$
\begin{aligned}
D_i(x,y) = & \frac{1}{1-x-y}\Bigg(\frac{1}{L_{1,h^0}}\,(m_i\,(x-1)+m_{l_1}\,x\,(y-1)) - \\
& \qquad - \frac{1}{L_{1,A^0}}(m_i\,(1-x)+m_{l_1}\,x\,(y-1))\Bigg) - \\
& -\Bigg(\frac{1}{L^2_{2,h^0}}(m_i\,(x^2-2\,x-2\,y+2\,x\,y)-2\,m_{l_1}\,x\,y\,(x+y)) - \\
& \quad - \frac{1}{L^2_{2,A^0}}\Big(m_i\,(x^2-2\,x-2\,y+2\,x\,y)+2\,m_{l_1}\,x\,y\,(x+y)\Big)\Bigg)\,\mathrm{a.k} - \\
& - \left(\frac{1}{L^2_{2,h^0}}(2x(m_i+m_{l_1}\,x)) - \frac{1}{L^2_{2,A^0}}\Big(2x(m_i-m_{l_1}x)\Big)\right)\mathrm{a.p.} + \\
& +\frac{1}{4}\left(\frac{1}{L^2_{2,h^0}}-\frac{1}{L^2_{2,A^0}}\right)m_i x(1-4\,x-4\,y)(\mathrm{b.k}-2\,\mathrm{b.p}) - \\
& -\Bigg\{\frac{f_6}{L^2_{1,h^0}}+\frac{f_5}{L^2_{1,A^0}}-2\,y\,\left(\frac{1}{L_{1,h^0}}\,(3\,m_i+4\,x\,m_{l_1})-\frac{1}{L_{1,A^0}}\,(3\,m_i-4\,x\,m_{l_1})\right) - \\
& \quad - \frac{f_8}{L^2_{2,h^0}}+\frac{f_7}{L^2_{2,A^0}}+2\frac{1}{L_{2,h^0}}(m_i\,(1-3\,x-3\,y)-m_{l_1}\,x)-2\,\frac{1}{L_{2,h^0}}\times \\
& \quad \times(m_i\,(1-3\,x-3\,y)+m_{l_1}\,x)\Bigg\}c^{\mathrm{Sym}} + \\
& \quad + \left(2\,\left(\frac{1}{L^2_{1,h^0}}-\frac{1}{L^2_{1,A^0}}\right)m_i xy + \left(\frac{1}{L^2_{2,h^0}}-\frac{1}{L^2_{2,A^0}}\right)m_i x^3\right)c^{\mathrm{ASym}}[p,p'] - \\
& \quad - \Bigg\{y\frac{1}{L^2_{1,h^0}}\Big(m_i^2(-1+x-y)+2m_i m_{l_1}x(-1+x+y) + \\
& \qquad\qquad + m_{l_1}^2 x(x+y)(x+y-1)\Big) + \\
& \qquad + y\,\frac{1}{L^2_{1,A^0}}\Big(m_i^2\,(1-x+y)+2\,m_i\,m_{l_1}\,x\,(-1+x+y) - \\
& \qquad\qquad - m_{l_1}^2\,x\,(x+y)\,(x+y-1)\,\Big) - \\
& \qquad - \left(\frac{1}{L_{1,h^0}}-\frac{1}{L_{1,A^0}}\right)\,y\,(4\,x+4\,y-3)+(x+y)\,\frac{1}{L^2_{2,h^0}}\times \\
& \qquad \times(m_i^2\,(1-x+y)+2\,m_i\,m_{l_1}\,x\,y+m_{l_1}^2\,x\,y\,(x+y-1))-(x+y)\times \\
& \qquad \times\,\frac{1}{L^2_{2,A^0}}\,(m_i^2\,(1-x+y)-2\,m_i\,m_{l_1}\,x\,y-m_{l_1}^2 x\,y\,(x+y-1)) + \\
& \qquad + (x+y)\left(\frac{1}{L_{2,h^0}}-\frac{1}{L_{2,A^0}}\right)(-3+4\,x+4\,y)\Bigg\}\mathrm{e.k} -
\end{aligned}
$$

$$
\begin{aligned}
&-\left\{x\frac{1}{L_{1,h^0}^2}(m_i^2\,(-1+x-y)-2m_i\,m_{l_1}x+m_{l_1}^2x(x+y)\,(-1+x+y))+\right.\\
&\quad+x\frac{1}{L_{1,A^0}^2}\left(m_i^2\,(1-x+y)-2m_im_{l_1}x+m_{l_1}^2x(x+y)\,(1-x-y)-\right.\\
&\qquad\qquad-\left(\frac{1}{L_{1,h^0}}-\frac{1}{L_{1,A^0}}\right)x(4x+4y-3)x-\frac{1}{L_{2,h^0}^2}\times\\
&\qquad\qquad\times(m_i^2(1-x+y)-2m_im_{l_1}x+m_{l_1}^2xy(x+y-1))+\\
&\qquad\qquad+\frac{1}{L_{2,A^0}^2}(m_i^2(1-x+y)-2m_im_{l_1}x+m_{l_1}^2xy(x+y-1))-\\
&\qquad\qquad\left.\left.-x\left(\frac{1}{L_{2,h^0}}-\frac{1}{L_{2,A^0}}\right)(-3+4x+4y)\right)\right\}\mathrm{e.p}-\\
&-2ix\left(\frac{1}{L_{2,h^0}^2}-\frac{1}{L_{2,A^0}^2}\right)(2y(y-1)+x(3y-1))h[p,p']-2i\times\\
&\times\left(\frac{1}{L_{1,h^0}^2}-\frac{1}{L_{1,A^0}^2}\right)x(x+y)(-2\,y^2\,g[p,p',k]+x\,(x-2\,y)\,g[p,p',p])+\\
&+i\left(\frac{1}{L_{1,h^0}}-\frac{1}{L_{1,A^0}}\right)x^2\,g[k,\beta,\beta]+i\left(\frac{1}{L_{2,h^0}^2}-\frac{1}{L_{2,A^0}^2}\right)\times\\
&\times(x(-4y(x+y)(x+y-1)g[p,p',k]+\\
&\quad+2(-2y+2y^2+3xy-x)\,g[p,p',p]))-\\
&\left.-i\left(\frac{1}{L_{2,h^0}}-\frac{1}{L_{2,A^0}}\right)x(1-x)g[k,\beta,\beta]\right\},
\end{aligned}
$$

$$
\begin{aligned}
D_i'(x,y)\;=\;&-\left\{\frac{1}{4}\frac{1}{L_{1,h^0}^2}(8\,m_{l_1}\,x\,y\,(-1+x+y)+m_i\,(4\,x^2-4\,x\,y+8\,y+x))+\right.\\
&\quad+\frac{1}{4}\frac{1}{L_{1,A^0}^2}(8\,m_{l_1}\,x\,y\,(-1+x+y)-m_i\,(4\,x^2-4\,x\,y+8\,y+x))+\\
&\quad+\frac{1}{L_{2,h^0}^2}(m_i\,(x^2+2\,y)+2m_{l_1}\,x\,y\,(x+y))+\\
&\quad\left.+\frac{1}{L_{2,A^0}^2}(-m_i(x^2+2\,y)+2\,m_{l_1}\,x\,y\,(x+y)\,)\right\}\mathrm{b.k}-\\
&-\left\{x\frac{1}{2}\frac{1}{L_{1,h^0}^2}(-4\,m_{l_1}x-m_i(3-4\,y))+\frac{1}{2}x\frac{1}{L_{1,A^0}^2}(-4m_{l_1}\,x+m_i\,(3-4\,y))-\right.\\
&\quad\left.-2x\,\left(\frac{1}{L_{2,h^0}^2}(m_i+m_{l_1}\,x)+\frac{1}{L_{2,A^0}^2}(-m_i+m_{l_1}\,x)\right)\right\}\mathrm{b.p}+\\
&+\left\{x\,y\,m_{l_1}^2\left(\frac{2}{L_{1,h^0}^2}(m_{l_1}\,x\,(x+y)-m_i\,(y-x))+\right.\right.
\end{aligned}
$$

$$
\begin{aligned}
&+ \frac{2}{L^2_{1,A^0}}(m_{l_1}\, x\,(x+y) - m_i\,(x-y))\Bigg) - \\
&- x(x+y)(y-1)m^2_{l_1}\left(\frac{1}{L^2_{2,h^0}}(m_i - 2m_{l_1}x) - \frac{1}{L^2_{1,A^0}}(m_i + 2m_{l_1}x)\right) - \\
&- 8\left(\frac{1}{L_{1,h^0}} + \frac{1}{L_{1,A^0}}\right) m_{l_1}xy - 2\left(\frac{1}{L_{2,h^0}} + \frac{1}{L_{2,A^0}}\right) m_{l_1}x \times \\
&\times (1-4x-4y)\Bigg\} d^{\rm Sym} - \\
&-\Bigg\{4\,xy^2\left(\frac{1}{L^2_{1,h^0}} - \frac{1}{L^2_{1,A^0}}\right) m_i + \left(\frac{1}{L^2_{2,h^0}} - \frac{1}{L^2_{2,A^0}}\right) m_i x \times \\
&\times (3x^2 + 2(y-1)y + x(5\,y-3))\Bigg\} \times \\
&\times d^{\rm ASym}[p,p'] - ixy^3\left(\frac{1}{L^2_{1,h^0}} - \frac{1}{L^2_{1,A^0}}\right) g[p,p',k]\,,
\end{aligned}
$$

$$
\begin{aligned}
E_{i\,,\mu}(x,y) \;=\; & m_{l_1}\Bigg\{\frac{1}{L^2_{2,h^0}}(-m^2_i\,(1+y) + m^2_{l_1}\,x\,y\,(x+y-1) + m_i\,m_{l_1}\,x\,(x+2y)) - \\
&- \frac{1}{L^2_{1,A^0}}(m^2_i(1+y) - m^2_{l_1}xy(x+y-1) - m_i\,m_{l_1}x(x+2y)) - \\
&- \left(\frac{1}{L_{2,h^0}} + \frac{1}{L_{2,A^0}}\right)(1+x+y)\Bigg\} a_\mu - \\
&-i\,\epsilon_{\rho\lambda\alpha\mu}\left(\frac{1}{L^2_{2,h^0}} - \frac{1}{L^2_{2,A^0}}\right) m_i\,x\,p_\rho\,p'_\lambda\,b_\alpha - \\
&-\Bigg\{\frac{1}{L^2_{1,h^0}}\,m_i y(m^2_i + m_i\,m_{l_1}\,x - m^2_{l_1}x(x-y)) + \\
&+ \frac{1}{L^2_{1,A^0}}(m_i\,m_{l_1}\,x - m^2_i + m^2_{l_1}\,x\,(x-y)) - \\
&- \frac{1}{L_{1,h^0}}\,y(m_i + 4\,m_{l_1}\,x - 2\,m_{l_1}\,y) + \frac{1}{L_{1,A^0}}(m_i - 4\,m_{l_1}\,x + 2\,m_{l_1}\,y) - \\
&- (x+y)\Bigg(\frac{1}{L^2_{2,h^0}}(3\,m^3_i + 3\,m^2_i\,m_{l_1}\,x + 3\,m^3_{l_1}\,x^2\,y + m_i\,m^2_{l_1} \times \\
&\times x\,(2-2\,x+y)) + \frac{1}{L^2_{2,A^0}} \times \\
&\times (-3\,m^3_i + 3\,m^2_i\,m_{l_1}x + 3m^3_{l_1}x^2\,y - \\
&- m_i m^2_{l_1}x(2-2x+y))\Bigg) -
\end{aligned}
$$

$$
\begin{aligned}
&- \frac{1}{L_{2,h^0}}(m_i\,(1+5\,x+5\,y) + m_{l_1}\,(6\,x^2+4\,x\,y-2\,y^2)) + \\
&+ \frac{1}{L_{2,A^0}}(m_i\,(1+5\,x+5\,y) - m_{l_1}\,(6\,x^2+4\,x\,y+2\,y^2))\Bigg\}\, c^{\mathrm{ASym}}[k,\mu] - \\
-\Bigg\{ &2m_i\left(\left(\frac{1}{L_{1,h^0}^2}-\frac{1}{L_{1,A^0}^2}\right) f_3 + \left(\frac{1}{L_{2,h^0}^2}-\frac{1}{L_{2,A^0}^2}\right) m_i\,m_{l_1}\,x\right) + \\
&+ 6\left(\frac{1}{L_{1,h^0}}-\frac{1}{L_{1,A^0}}\right) m_i\,y - \frac{1}{L_{2,h^0}^2} \times \\
&\times \Big(2\,m_i^3\,(x+y) + 2\,m_{l_1}\,m_i^2\,x\,(x+y) + 2\,m_{l_1}^3\,x^2\,y\,(x+y) - \\
&\quad - m_{l_1}^2\,m_i\,x\,(x^2-2\,x\,(y-1)-2\,y^2)\Big) - \\
&- \frac{1}{L_{2,A^0}^2}(-2\,m_i^3\,(x+y) + 2\,m_{l_1}\,m_i^2\,x\,(x+y) + \\
&\qquad\qquad + 2\,m_{l_1}^3\,x^2\,y\,(x+y) + m_{l_1}^2\,x\,m_i\,(x^2-2\,x\,(y-1)-2\,y^2)) - \\
&- 2\frac{1}{L_{2,h^0}}(\,m_i\,(1+3\,x+3\,y) + m_{l_1}\,(4\,x^2+4\,x\,y)) + \\
&+ 2\frac{1}{L_{2,A^0}}(m_i\,(1+3\,x+3\,y) - m_{l_1}\,(4\,x^2+4\,x\,y))\Bigg\} c^{\mathrm{ASym}}[p,\mu] - \\
-\Bigg\{ &m_{l_1}\,m_i\,y\left(\left(\frac{1}{L_{1,h^0}^2}+\frac{1}{L_{1,A^0}^2}\right) f_2 + 2\left(\frac{1}{L_{1,h^0}^2}-\frac{1}{L_{1,A^0}^2}\right) m_{l_1}\,m_i\,x\right) + \\
&+ \left(\frac{1}{L_{1,h^0}}+\frac{1}{L_{1,A^0}}\right) m_{l_1}\,m_i\,y - m_{l_1} \times \\
&\times \Bigg(\frac{1}{L_{2,h^0}}(m_i\,(x+y-2) - 4\,m_{l_1}\,x\,(x+y)\,) + \\
&\quad + \frac{1}{L_{2,A^0}}(m_i\,(x+y-2) + 4\,m_{l_1}\,x\,(x+y))\Bigg)\Bigg\}\, e_\mu - \\
-i\Bigg\{ &\frac{1}{2}\,\frac{1}{L_{1,h^0}^2}\,(m_i^2\,(1-x-y) + m_{l_1}^2\,(x^2-x^3+x\,y^2) + m_i\,m_{l_1}\,x) + \\
&+ \frac{1}{2}\frac{1}{L_{1,A^0}^2}(m_i^2(-1+x+y) + m_{l_1}^2(-x^2+x^3-xy^2) + m_i m_{l_1} x) + \\
&+ \frac{1}{2}\left(\frac{1}{L_{1,h^0}}-\frac{1}{L_{1,A^0}}\right)(x-3y) - 2\frac{1}{L_{2,h^0}^2}(m_i^2(y-1) + m_{l_1}^2 xy + m_i m_{l_1} y) - \\
&- 2\frac{1}{L_{2,h^0}^2}\,(m_i^2\,(1-y) - m_{l_1}^2\,x\,y + m_i\,m_{l_1}\,y\,) - \\
&- 2\,\left(\frac{1}{L_{2,h^0}}-\frac{1}{L_{2,A^0}}\right)\,(3\,x+2\,y)\Bigg\}\, h[k,\mu] +
\end{aligned}
$$

$$
\begin{aligned}
&+ i\Bigg\{ 2\, \frac{1}{L^2_{2,h^0}}\, (f_4 + m_{l_1}\, x\, (m_i + m_{l_1}\, x\, y)) + \\
&\quad + 2\, \frac{1}{L^2_{2,A^0}} (-f_4 + m_{l_1}\, x\, (m_i - m_{l_1}\, x\, y)) - \\
&\quad - 2\, \left(\frac{1}{L_{2,h^0}} - \frac{1}{L_{2,A^0}} \right)\, (x+y) \Bigg\} h[p,\mu] - \epsilon_{\rho\theta\alpha\mu}\, \left(\frac{1}{L^2_{2,h^0}} + \frac{1}{L^2_{2,A^0}} \right) x^2 \times \\
&\times k_\rho\, p_\theta\, h[p,\alpha] - 2\, i\, \left(\frac{1}{L^2_{1,h^0}} - \frac{1}{L^2_{1,A^0}} \right)\, m^2_{l_1}\, x\, (x+y) \times \\
&\times (y^2\, g[\mu,k,k] + x\, y\, g[\mu,k,p] + x\, y\, g[\mu,p,k] + x^2\, g[\mu,p,p]) + \\
&+ \frac{i}{2}\, \left(\frac{1}{L_{1,h^0}} - \frac{1}{L_{1,A^0}} \right) \times \\
&\times (4\, (2+3\, x)\, y\, g[k,p,\mu] + 4y(1+x+y) g[\mu,k,k] + 4x(x+y) g[\mu,k,p] - \\
&\quad - 8(y-3y) g[\mu,p,k] + 24xy g[\mu,p,p] + 2x^2 m^2_{l_1} g[\mu,\beta,\beta]) + \\
&+ \frac{i}{2} \frac{1}{L^2_{2,h^0}} ((x+y)\, (m_i\, (-1+x+y) - 4\, m_{l_1}\, (y-1)\, (x+y)\, g[\mu,k,k] + \\
&\qquad\qquad + 4\, m_{l_1}\, x\, (y-1)\, ((x+y)\, g[\mu,k,p] - (x+y)\, g[\mu,p,k] + \\
&\qquad\qquad + x\, g[\mu,p,p]))) + \frac{i}{2}\, \frac{1}{L^2_{2,A^0}} \times \\
&\times ((x+y)(m_i\, (-1+x+y) + 4\, m_{l_1}\, (y-1)\, (x+y)\, g[\mu,k,k] - \\
&\qquad - 4 m_{l_1}\, x\, (y-1)\, ((x+y)\, g[\mu,k,p] - (x+y)\, g[\mu,p,k] + \\
&\qquad + x\, g[\mu,p,p]))) - i \left(\frac{1}{L_{2,h^0}} - \frac{1}{L_{2,A^0}} \right) \times \\
&\times (2(x + 3x^2 - 2y^2 + 2y) g[p,p',\mu] + 4\, (-2\, x + 4\, x^2 - 2\, y + 7xy + 3y^2) \times \\
&\quad \times\, g[\mu,k,k] + 2\, x\, (3 - 8\, x - 7\, y)\, g[\mu,k,p] + \\
&\quad + 2\, (4\, x - 6\, x^2 + 4\, y - 9\, x\, y - 4\, y^2) g[\mu,p,k] - 4\, x\, (2 - 3\, x - 3\, y)\, \times \\
&\quad \times g[\mu,p,p] - m^2_{l_1}\, x\, (x+1)\, g[\mu,\beta,\beta])\, ,
\end{aligned}
$$

$$
\begin{aligned}
E'_{i\,,\mu}(x,y) \,=\, & i\, \epsilon_{\rho\theta\alpha\mu}\, \left(\frac{1}{L^2_{2,h^0}} - \frac{1}{L^2_{2,A^0}} \right)\, m_i\, x\, k_\rho\, p_\theta\, a_\alpha + \\
& + \Bigg\{ -m_{l_1} \Bigg(2\, \left(\frac{1}{L^2_{1,h^0}} - \frac{1}{L^2_{2,A^0}} \right) m_{l_1}\, m_i\, (x-1)\, x\, + \\
&\qquad + \left(\frac{1}{L^2_{1,h^0}} + \frac{1}{L^2_{2,A^0}} \right)\, f_2\, (-1+x+y) \Bigg) + \left(\frac{1}{L_{1,h^0}} + \frac{1}{L_{1,A^0}} \right) \times \\
&\quad \times\, m_{l_1}\, (-1+3x-y) - \left(\frac{1}{L^2_{2,h^0}} + \frac{1}{L^2_{2,A^0}} \right)\, m_{l_1}\, f_4 -
\end{aligned}
$$

$$
\begin{aligned}
&\quad -\left(\frac{1}{L_{2,h^0}^2}-\frac{1}{L_{2,A^0}^2}\right) m_{l_1}^2 m_i x^2+\left(\frac{1}{L_{2,h^0}}+\frac{1}{L_{2,A^0}}\right) m_{l_1}(1+x+y)\Bigg\} b_\mu - \\
&-\Bigg\{2\left(\frac{1}{L_{1,h^0}^2}-\frac{1}{L_{1,A^0}^2}\right) m_{l_1}^2 m_i x y^2+2\left(\frac{1}{L_{1,h^0}}+\frac{1}{L_{1,A^0}}\right) m_{l_1}(2x-y)y - \\
&\quad -2\left(\frac{1}{L_{2,h^0}}+\frac{1}{L_{2,A^0}}\right) m_{l_1}\left(3x^2+4xy+y^2\right)\Bigg\} d^{\mathrm{ASym}}[k,\mu] + \\
&+\Bigg\{4\left(\frac{1}{L_{1,h^0}^2}-\frac{1}{L_{1,A^0}^2}\right) m_{l_1}^2\, m_i\, x\, y^2+\left(\frac{1}{L_{2,h^0}^2}-\frac{1}{L_{2,A^0}^2}\right) m_{l_1}^2\, m_i\, x\, \times \\
&\quad \times\left(3x^2+2(y-1)\,y+x\,(5y-3)\right)\Bigg\} d^{\mathrm{ASym}}[p,\mu] - \\
&-i\Bigg\{\left(\frac{1}{L_{1,h^0}^2}+\frac{1}{L_{1,A^0}^2}\right) m_{l_1} m_i y\left(m_{l_1}^2 x(x+y)-m_i^2\right)-3\left(\frac{1}{L_{1,h^0}}+\frac{1}{L_{1,A^0}}\right)\times \\
&\quad \times m_{l_1}\, m_i\, y-m_{l_1}\left(\frac{1}{L_{2,h^0}^2}\left(m_{l_1}x+m_i\right)+\frac{1}{L_{2,A^0}^2}\left(-m_{l_1}x+m_i\right)\right)\times \\
&\quad \times (x+y)\left(m_{l_1}^2\, x\, y+m_i^2\right)-(x+y)\, m_{l_1}\times \\
&\quad \times\left(3\left(\frac{1}{L_{2,h^0}}+\frac{1}{L_{2,A^0}}\right) m_i+4\left(\frac{1}{L_{2,h^0}}-\frac{1}{L_{2,A^0}}\right) m_{l_1}\, x\right)\Bigg\}\times \\
&\times f_\mu+2\,x\,\epsilon_{\rho\theta\alpha\mu}\left(\frac{1}{L_{1,h^0}^2}-\frac{1}{L_{1,A^0}^2}\right)\left(y\,h[k,\alpha]+x\,h[p,\alpha]\right) k_\rho\, p_\theta+\epsilon_{\rho\alpha\beta\mu}\times \\
&\times\Bigg\{\frac{1}{2}\left(\frac{1}{L_{1,h^0}^2}-\frac{1}{L_{1,A^0}^2}\right)\left(m_i^2\,(-1+x-y)+m_{l_1}^2\, x\,(x+y)\,(x+y-1)\right)k_\rho - \\
&\quad -\frac{1}{2}\left(\frac{1}{L_{1,h^0}^2}+\frac{1}{L_{1,A^0}^2}\right) x\, m_i\, m_{l_1}\, p_\rho-\frac{3}{2}\left(\frac{1}{L_{1,h^0}}-\frac{1}{L_{1,A^0}}\right)(x+y)\, k_\rho - \\
&\quad -\frac{1}{2}\frac{1}{L_{2,h^0}^2}\left(2\, m_i\, m_{l_1}\, x+m_i^2\,(y+1)+m_{l_1}^2\, x\, y\,(-1+x+y)\right) k_\rho + \\
&\quad +\frac{1}{2}\frac{1}{L_{2,A^0}^2}\left(-2\, m_i\, m_{l_1} x+m_i^2\,(y+1)+m_{l_1}^2\, x\, y\,(-1+x+y)\right)k_\rho - \\
&\quad -\frac{3}{2}\left(\frac{1}{L_{2,h^0}}-\frac{1}{L_{2,A^0}}\right)(x+y)k_\rho\Bigg\} h[\alpha,\beta]-i\left(\frac{1}{L_{1,h^0}^2}-\frac{1}{L_{1,A^0}^2}\right)\times \\
&\times m_{l_1}^2\, x\, y^3\, g[\mu,p,k]+\epsilon_{\rho\alpha\beta\mu}\times \\
&\times\Bigg(\frac{1}{2}\left(\frac{1}{L_{1,h^0}}-\frac{1}{L_{1,A^0}}\right) xy\left(g[k,\alpha,\beta]+2g[\beta,\alpha,k]\right) - \\
&\quad -\frac{1}{2}\left(\frac{1}{L_{2,h^0}^2}+\frac{1}{L_{2,A^0}^2}\right) m_i\, m_{l_1}\, x^2\,(x+y)\, g[\beta,\alpha,k] -
\end{aligned}
$$

$$-\frac{1}{2}\left(\frac{1}{L_{2,h^0}}-\frac{1}{L_{2,A^0}}\right)x\,(x+y)\,(g[k,\alpha,\beta]+2\,g[\beta,\alpha,k])\Bigg)\,k_\rho\,, \tag{A.1}$$

where

$$\begin{aligned}
L_{1,S} &= m_S^2\,(x+(1-x)\,z_h+x\,(-1+x+y)\,z_{1,h})\,,\\
L_{2,S} &= m_S^2\,(x+(1-x)\,z_h-x\,y\,z_{1,h})\,,\\
f_1 &= m_i^2\,(y+1)-m_{l_1}^2\,x\,y\,(x+y-1)\,,\\
f_2 &= m_i^2+m_{l_1}^2\,x\,(x+y)\,,\\
f_3 &= m_i^2-m_{l_1}^2\,x\,(x+y)\,,\\
f_4 &= m_i^2\,(1-y)+m_{l_1}^2\,x\,y\,(x+y-1)\,,\\
f_5 &= m_i\,(f_3+m_i^2)\,y+m_{l_1}x\,\times\\
&\quad\times\left(2\,m_{l_1}^2\,x\,(y-1)\,(x+y)-2\,m_i^2\,y-m_i\,m_{l_1}^2\,(2\,x^2+y^2+x\,y-y)\right)\,,\\
f_6 &= -m_i\,(f_3+m_i^2)\,y+m_{l_1}x\,\times\\
&\quad\times\left(2\,m_{l_1}^2\,x\,(y-1)\,(x+y)-2\,m_i^2\,y+m_i\,m_{l_1}^2\,(2\,x^2+y^2+x\,y-y)\right)\,,\\
f_7 &= 2\,m_i^3\,(x+y)-2\,m_{l_1}^3\,x^3+m_{l_1}\,m_i\,x\,(-2\,m_i\,(x+y)+m_{l_1}\,(x+2\,x\,y+2\,y^2-2\,y))\,,\\
f_8 &= 2\,m_i^3\,(x+y)+2\,m_{l_1}^3\,x^3+m_{l_1}\,m_i\,x\,(2\,m_i\,(x+y)+m_{l_1}\,(x+2\,x\,y+2\,y^2-2\,y))\,,
\end{aligned} \tag{A.2}$$

with $z_h = m_i^2/m_S^2$, $z_{1,h} = m_{l_1}^2/m_S^2$ and for $S = h^0, A^0$. Here we use $h[r,r'] = h_{\alpha\beta}\,r^\alpha\,r'^\beta$, $g[r,r',r''] = g_{\alpha\beta\gamma}\,r^\alpha\,r'^\beta\,r''^\gamma$ and parametrize the coefficients $c[r,r']$ and $d[r,r']$ as $c[r,r'] = c^{Asym}_{\alpha\beta}\,r^\alpha\,r'^\beta + c^{\rm Sym}\,r.r'$, $d[r,r'] = d^{Asym}_{\alpha\beta}\,r^\alpha\,r'^\beta + d^{\rm Sym}\,r.r'$, where $Asym$ (Sym) denotes the asymmetry (symmetry) in the indices. Here $h_{\alpha\beta}$ ($g_{\alpha\beta\gamma}$) is taken antisymmetric with respect to indices (first two indices) and $d_{\alpha\beta}$ ($c_{\alpha\beta}$) traceless.

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References

[1] V.A. Kostelecký ed., *CPT and Lorentz Symmetry*, World Scientific, Singapore, 1999.

[2] V.A. Kostelecký and S. Samuel, *Spontaneous breaking of Lorentz symmetry in string theory*, *Phys. Rev.* **D 39** (1989) 683; *Gravitational phenomenology in higher dimensional theories and strings*, *Phys. Rev.* **D 40** (1989) 1886; *Phenomenological gravitational constraints on strings and higher dimensional theories*, *Phys. Rev. Lett.* **63** (1989) 224; *Photon and graviton masses in string theories*, *Phys. Rev. Lett.* **66** (1991) 1811;
V.A. Kostelecký and R. Potting, *Expectation values, Lorentz invariance and CPT in the open bosonic string*, *Phys. Lett.* **B 381** (1996) 89 [hep-th/9605088]; *Analytical construction of a nonperturbative vacuum for the open bosonic string*, *Phys. Rev.* **D 63** (2001) 046007 [hep-th/0008252];
V.A. Kostelecký, M. Perry and R. Potting, *Off-shell structure of the string sigma model*, *Phys. Rev. Lett.* **84** (2000) 4541 [hep-th/9912243].

[3] S.M. Carroll J.A. Harvey, V.A. Kostelecký, C.D. Lane and T. Okamoto, *Noncommutative field theory and Lorentz violation*, *Phys. Rev. Lett.* **87** (2001) 141601 [hep-th/0105082];
Z. Guralnik, R. Jackiw, S.Y. Pi and A.P. Polychronakos, *Testing non-commutative QED, constructing non-commutative MHD*, *Phys. Lett.* **B 517** (2001) 450 [hep-th/0106044];
A. Anisimov, T. Banks, M. Dine and M. Graesser, *Comments on non-commutative phenomenology*, *Phys. Rev.* **D 65** (2002) 085032 [hep-ph/0106356];
C.E. Carlson, C.D. Carone and R.F. Lebed, *Bounding noncommutative QCD*, *Phys. Lett.* **B 518** (2001) 201 [hep-ph/0107291].

[4] A. Kostelecký, R. Lehnert and M. Perry, *Spacetime-varying couplings and Lorentz violation*, astro-ph/0212003.

[5] G. Barenboim, L. Borissov and J. Lykken, *Neutrinos that violate CPT and the experiments that love them*, *Phys. Lett.* **B 534** (2002) 106 [hep-ph/0201080];
J. Bernabeu, *CP, T violation in neutrino oscillations*, in *Proc. Weak Interactions and Neutrinos WIN'99*, Cape Town, South Africa, 24–30 Jan 1999 [hep-ph/9904474];
M. Jacobson and T. Ohlsson, *Extrinsic cpt violation in neutrino oscillations in matter*, hep-ph/0305064.

[6] D. Colladay and V.A. Kostelecký, *CPT violation and the standard model*, *Phys. Rev.* **D 55** (1997) 6760 [hep-ph/9703464]; *Lorentz-violating extension of the standard model*, *Phys. Rev.* **D 58** (1998) 116002 [hep-ph/9809521]; *Cross sections and Lorentz violation*, *Phys. Lett.* **B 511** (2001) 209 [hep-ph/0104300].

[7] V.A. Kostelecký and R. Lehnert, *Stability, causality and Lorentz and CPT violation*, *Phys. Rev.* **D 63** (2001) 065008 [hep-th/0012060].

[8] KTeV collaboration, Y.B. Hsiung, *An observation of direct CP-violation — ϵ'/ϵ result from KTeV*, *Nucl. Phys.* **86** *(Proc. Suppl.)* (2000) 312.

[9] OPAL collaboration, K. Ackerstaff et al., *A study of B meson oscillations using hadronic Z^0 decays containing leptons*, *Z. Physik* **C 76** (1997) 401 [hep-ex/9707009];
DELPHI collaboration, M. Feint et al., prep. DELPHI 97-98 CONF 80, 1997
Belle collaboration, K. Abe et al., *Measurement of B_d^0-$\bar{B}_d^0$ mixing rate from the time evolution of dilepton events at the $\Upsilon(4S)$*, *Phys. Rev. Lett.* **86** (2001) 3228 [hep-ex/0011090].

[10] V.A. Kostelecký and R. Potting, *CPT, strings and meson factories*, *Phys. Rev.* **D 51** (1995) 3923 [hep-ph/9501341];
D. Colladay and V.A. Kostelecký, *Tests of direct and indirect CPT violation at a B factory*, *Phys. Lett.* **B 344** (1995) 259 [hep-ph/9501372]; *Testing CPT with the neutral-d system*, *Phys. Rev.* **D 52** (1995) 6224 [hep-ph/9510365];
V.A. Kostelecký, and R. Van Kooten, *Bounding CPT violation in the neutral B system*, *Phys. Rev.* **D 54** (1996) 5585 [hep-ph/9607449];
V.A. Kostelecký, *Sensitivity of CPT tests with neutral mesons*, *Phys. Rev. Lett.* **80** (1998) 1818 [hep-ph/9809572]; *Signals for CPT and Lorentz violation in neutral meson oscillations*, *Phys. Rev.* **D 61** (2000) 016002 [hep-ph/9909554]; *Formalism for CPT, T, and Lorentz violation in neutral meson oscillations*, *Phys. Rev.* **D 64** (2001) 076001 [hep-ph/0104120];
N. Isgur et al., *Background enhancement of CPT reach at an asymmetric ϕ factory*, *Phys. Lett.* **B 515** (2001) 333 [hep-ph/0106353];
O. Bertolami et al., *CPT violation and baryogenesis*, *Phys. Lett.* **B 395** (1997) 178 [hep-ph/9612437].

JHEP06(2003)016

[11] D. Bear, R.E. Stoner, R.L. Walsworth, V.A. Kostelecký and C.D. Lane, *Limit on Lorentz and CPT violation of the neutron using a two-species noble-gas maser*, *Phys. Rev. Lett.* **85** (2000) 5038 [physics/0007049];
D.F. Phillips et al., *Limit on Lorentz and CPT violation of the proton using a hydrogen maser*, *Phys. Rev.* **D 63** (2001) 111101 [physics/0008230];
M.A. Humphrey, D.F. Phillips and R.L. Walsworth, *Testing Lorentz and CPT symmetry with hydrogen masers*, physics/0103068; *Double-resonance frequency shift in a hydrogen maser*, *Phys. Rev.* **A 62** (2000) 063405;
V.A. Kostelecký and C.D. Lane, *Constraints on Lorentz violation from clock-comparison experiments*, *Phys. Rev.* **D 60** (1999) 116010 [hep-ph/9908504]; *Nonrelativistic quantum hamiltonian for Lorentz violation*, *J. Math. Phys.* **40** (1999) 6245 [hep-ph/9909542];
R. Bluhm, V.A. Kostelecký, C.D. Lane and N. Russell, *Clock-comparison tests of Lorentz and CPT symmetry in space*, *Phys. Rev. Lett.* **88** (2002) 090801 [hep-ph/0111141].

[12] H. Dehmelt, R. Mittleman, J. van Dyck, R.S. and P. Schwinberg, *Past electron positron $g-2$ experiments yielded sharpest bound on CPT violation*, *Phys. Rev. Lett.* **83** (1999) 4694 [hep-ph/9906262];
R.K. Mittleman, I.I. Ioannou, H.G. Dehmelt and N. Russell, *Bound on CPT and Lorentz symmetry with a trapped electron*, *Phys. Rev. Lett.* **83** (1999) 2116;
G. Gabrielse et al., *Precision mass spectroscopy of the antiproton and proton using simultaneously trapped particles*, *Phys. Rev. Lett.* **82** (1999) 3198;
R. Bluhm, V.A. Kostelecký and N. Russell, *CPT and Lorentz tests in hydrogen and antihydrogen*, *Phys. Rev. Lett.* **82** (1999) 2254 [hep-ph/9810269]; *Testing CPT with anomalous magnetic moments*, *Phys. Rev. Lett.* **79** (1997) 1432 [hep-ph/9707364];
R. Bluhm and V.A. Kostelecký, *Lorentz and CPT tests with spin polarized solids*, *Phys. Rev. Lett.* **84** (2000) 1381 [hep-ph/9912542].

[13] R. Bluhm, V.A. Kostelecký and N. Russell, *CPT and Lorentz tests in penning traps*, *Phys. Rev.* **D 57** (1998) 3932 [hep-ph/9809543].

[14] S.M. Carroll, G.B. Field and R. Jackiw, *Limits on a Lorentz and parity violating modification of electrodynamics*, *Phys. Rev.* **D 41** (1990) 1231;
V.A. Kostelecký and M. Mewes, *Cosmological constraints on Lorentz violation in electrodynamics*, *Phys. Rev. Lett.* **87** (2001) 251304 [hep-ph/0111026].

[15] V.W. Hughes et al., *Test of CPT and Lorentz invariance from muonium spectroscopy*, *Phys. Rev. Lett.* **87** (2001) 111804 [hep-ex/0106103];
R. Bluhm, V.A. Kostelecký and C. Lane, *CPT and Lorentz tests with muons*, *Phys. Rev. Lett.* **84** (2000) 1098 [hep-ph/9912451].

[16] V.A. Kostelecký and A.G.M. Pickering, *Vacuum photon splitting in Lorentz-violating quantum electrodynamics*, hep-ph/0212382.

[17] V.A. Kostelecký, C.D. Lane and A.G.M. Pickerng, *One loop renormalization of Lorentz violating electrodynamics*, *Phys. Rev.* **D 65** (2002) 056006 [hep-th/0111123].

[18] D. Chang, W.S. Hou and W.Y. Keung, *Two loop contributions of flavor changing neutral Higgs bosons to $\mu \to e\gamma$*, *Phys. Rev.* **D 48** (1993) 217 [hep-ph/9302267].

[19] E.O. Iltan, *Electric dipole moments of charged leptons and lepton flavor violating interactions in the general two Higgs doublet model*, *Phys. Rev.* **D 64** (2001) 013013 [hep-ph/0101017].

[20] E.O. Iltan, *CP-violation in the lepton flavor violating interactions $\mu \to e\gamma$ and $\tau \to \mu\gamma$*, *Phys. Rev.* **D 64** (2001) 115005 [hep-ph/0107107].

[21] R. Diaz, R. Martinez and J.A. Rodriguez, *Lepton flavor violation in the two Higgs doublet model type-III*, *Phys. Rev.* **D 63** (2001) 095007 [hep-ph/0010149].

[22] R. Barbieri and L.J. Hall, *Signals for supersymmetric unification*, *Phys. Lett.* **B 338** (1994) 212 [hep-ph/9408406].

[23] R. Barbieri, L.J. Hall and A. Strumia, *Violations of lepton flavor and CP in supersymmetric unified theories*, *Nucl. Phys.* **B 445** (1995) 219 [hep-ph/9501334].

[24] R. Barbieri, L.J. Hall and A. Strumia, *Hadronic flavor and CP-violating signals of superunification*, *Nucl. Phys.* **B 449** (1995) 437 [hep-ph/9504373].

[25] P. Ciafaloni, A. Romanino and A. Strumia, *Lepton flavor violations in* SO(10) *with largge* $\tan\beta$, IFUP-YH-42-95.

[26] T.V. Duong, B. Dutta and E. Keith, *Mutual cosideration of* $b \to s\gamma$ *and* $\mu \to e\gamma$ *in supersymmetric* SO(10) *grand unification*, *Phys. Lett.* **B 378** (1996) 128 [hep-ph/9510441].

[27] G. Couture, M. Frank, H. Konig and M. Pospelov, $\mu \to e\gamma$ *decay in the left-right supersymmetric model*, *Eur. Phys. J.* **C 7** (1999) 135 [hep-ph/9701299].

[28] Y. Okada, K.-i. Okumura and Y. Shimizu, $\mu \to e\gamma$ *and* $\mu \to 3e$ *processes with polarized muons and supersymmetric grand unified theories*, *Phys. Rev.* **D 61** (2000) 094001 [hep-ph/9906446].

[29] MEGA collaboration, M.L. Brooks et al., *New limit for the family-number non-conserving decay* $\mu^+ \to e^+\gamma$, *Phys. Rev. Lett.* **83** (1999) 1521 [hep-ex/9905013].

[30] CLEO collaboration, S. Ahmed et al., *Update of the search for the neutrinoless decay* $\tau \to \mu\gamma$, *Phys. Rev.* **D 61** (2000) 071101 [hep-ex/9910060].

[31] E. Iltan and H. Sundu, *Anomalous magnetic moment of the muon in the two Higgs doublet model*, *Acta Phys. Slovaca* **53** (2003) 17.

[32] E.O. Iltan and H. Sundu, *The* $H^+ \to W^+ l_i^- l_j^+$ *decay in the two Higgs doublet model*, *Phys. Rev.* **D 67** (2003) 035006 [hep-ph/0211343].

[33] K. Abdullah, C. Carlberg, E.D. Commins, H. Gould and S.B. Ross, *A new experimental limit on the electron electric dipole moment*, *Phys. Rev. Lett.* **65** (1990) 2347.

[34] MEGA collaboration, M.L. Brooks et al., *New limit for the family-number non-conserving decay* $\mu^+ \to e^+\gamma$, *Phys. Rev. Lett.* **83** (1999) 1521 [hep-ex/9905013].